

# Large $N_c$ Expansion in Chiral Quark Model of Mesons

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## Abstract

We study  $SU(3)_L \times SU(3)_R$  chiral quark model of mesons up to the next to leading order of  $1/N_c$  expansion. Composite vector and axial-vector mesons resonances are introduced via non-linear realization of chiral  $SU(3)$  and vector meson dominant. Effects of one-loop graphs of pseudoscalar, vector and axial-vector mesons is calculated systematically and the significant results are obtained. We also investigate correction of quark-gluon coupling and relationship between chiral quark model and QCD sum rules. Up to powers four of derivatives, chiral effective lagrangian of mesons is derived and evaluated to the next to leading order of  $1/N_c$ . Low energy limit of the model is examined. Ten low energy coupling constants  $L_i (i = 1, 2, \dots, 10)$  in ChPT are obtained and agree with ChPT well.

## 1 Introduction

The Chiral Quark Model (ChQM) and its extensions[1]-[7] have attracted much interest continually during the last two decades. The reasons why the ChQM is so attractive are both because of its elegant descriptions of the features of low-energy QCD and because of its great successes in the different aspects of phenomenological predictions on hadron physics (see the brief review on ChQM in section 2 of this paper). However, so far, the studies on ChQM face two challenges as soon as we start going beyond lowest order: 1) How to go beyond the chiral perturbative theory(ChPT)? In order to capture physics between perturbative QCD and ChPT[8]-[10], freedoms of meson resonances have to be included into dynamics. It was well known that, in this energy region, there are no well-defined method to yield a convergence expansion. Therefore, we need some phenomenological models including spin-1 meson resonances as well pseudoscalar meson to describe the physics in this energy region. Although there are some discussion[5, 12], role of mesons resonances in ChQM are still uncertain. 2) How to go beyond large  $N_c$  limit? The ChQM-studies are limited to be on the level to catch merely the leading order effects of large  $N_c$  expansion. In other words, like other QCD-inspired models (e.g., bag model, Skyrme model, pole model and so on), only the leading order evaluations in ChQM are legitimate and practicable, and there is no way to calculate the next to leading order contributions in the model so far, even though there are

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some arguments on suppression of  $1/N_c$  in ChQM. This is a bad shortage. It makes the calculations of ChQM uncontrolled approximations, in that there is no well-defined way to put error bars on the predictions. In this sense, any conclusions coming from ChQM would become uncertain. In order to improve this unhappy situation of ChQM, it is necessary and urgent to study contributions beyond the leading order in ChQM. With these motivation, in this paper, we first extend ChQM to include spin-1 meson resonances. Then we like to provide a systematical study to illustrate the ChQM-calculations to be legitimate up to the next to leading order of  $1/N_c$  expansion.

The simplest version of ChQM which was originated by Weinberg[1], and developed by Manohar and Georgi[2] provides a QCD-inspired description on the simple constituent quark model(M-G model). In view of this model, in the energy region between the chiral symmetry-breaking scale ( $\Lambda_{CSSB} \simeq 2\pi f_\pi \simeq 1.2GeV$ ) and the confinement scale ( $\Lambda_{QCD} \sim 0.1 - 0.3GeV$ ), the dynamical field degrees of freedom are constituent quarks(quasi-particle of quarks), gluons and Goldstone bosons associated with Chiral Symmetry Spontaneous Breaking(CSSB). In this quasiparticle description, the effective coupling between gluon and quarks is small and the important interaction is the coupling between quarks and Goldstone bosons. Fields of pseudoscalar mesons in M-G model are treated as composite fields of quarks and anti-quarks instead of independent dynamical freedoms. Dynamics of pseudoscalar mesons is described by an effective Lagrangian, which is derived via integrating out the quark fields("freeze" the quark-freedom).

There are no meson resonances in original M-G model. From viewpoint of chiral symmetry, role of the lowest meson resonances in low energy effective theories has been analyzed systematically in Ref.[11]. The authors illustrate that all meson resonance fields can be treated on same level: they carry non-linear realizations of chiral  $SU(3)$  which are uniquely determined by the known transformation properties under the vectorial subgroup  $SU(3)_V$ (octets and singlet). This is an attractive symmetry property on meson resonances and shall be adopted in this present paper. Of course, it is not necessary to describe the degrees of freedom of vector and axial-vector mesons by antisymmetric tensor fields[11], and there are other phenomenological successful attempts to introduce spin-1 meson resonances as massive Yang-Mills fields[6, 12, 13]. In addition, it was well known that vector meson dominant(VMD) is a very important phenomenological feature in electro-weak interaction of hadrons. In ChQM, it is nature to introduce spin-1 meson resonances via VMD and is especially convenient to describe relevant degrees of freedom in terms of familiar vector representation(see section 2). Similar to M-G model, all freedoms of meson resonances in ChQM are treated as composite fields of quark pairs, effective lagrangian describing dynamics of mesons is derived via effects of quark loops.

According to large  $N_c$  argument[14], the color coupling between quarks and gluon fields also yields the leading order contribution at large  $N_c$  limit. However, this contribution was omitted in original M-G model and its many extensions. This problem has been first noticed in ref.[15], in that reference the authors investigated the effects of leading gluon coupling( $O(\alpha_s)$ ) in M-G model. Since it is unknown how to perform analytically the remaining integration over the gluon fields and this coupling become rather unclear in presence of meson composite fields, the authors suggested a possible way, which is to parameterize phenomenologically this effective coupling and make a weak gluon-field expansion around the resulting physical vacuum. In this present paper, in order to carry out a complete study on  $1/N_c$  expansion at leading order, the investigation in ref.[15] will be extended to include pseudoscalar, vector and axial-vector mesons. In ref.[15], the authors showed

that the gluon coupling contributes to low energy coupling constants  $L_3$ ,  $L_5$ ,  $L_8$  and  $L_{10}$  but does not contribute to other coupling constants of ChPT at  $O(p^4)$ . In particular, the correction due to gluon coupling contributes to  $L_3$  and  $L_{10}$  is about 50%. It makes  $L_3$  agree with experiment data. This is an important improvement so that the quark-gluon coupling in ChQM can not be omitted simply. From these results, it seems to be little fortunate ingredients in phenomenological success of those chiral quark model without the gluon coupling. However, in this paper we will show that, in case of presence of spin-1 meson resonances, the contribution of the gluon coupling is suppressed due to mixing between axial-vector and pseudoscalar mesons. The results are that gluon coupling corrects to  $L_{10}$  about 10% and to  $L_3$  about 3% only. Simultaneously,  $L_3$  and  $L_{10}$  are compensated by effect of exchange of spin-1 meson resonances and, hence, they agree with data well. Therefore, the gluon correction in presence of meson resonances is much smaller than one obtained in ref.[15]. Thus, it becomes legitimate to treat quark-gluon coupling as perturbation in the present model.

ChQM is a non-renormalizable effective field theory, since there are too few free parameters to introduce enough counterterms to absorb divergences from quark-loop and meson-loop integral. Therefore, we have to parameterize the divergences from quark loops as well as meson loops, i.e., we need to introduce cut-offs for regularizing these divergences. We first discuss the divergences from quark loops. From viewpoint of QCD, these divergences are caused by “point particle” approximation of mesons. Hence the cut-offs to regularize these divergences can be treated as description on scale of interaction of quark pairs in mesons. In this paper, corresponding to spin zero and one, two cut-offs are needed to describe internal interaction scale of  $0^-$  and  $1^\pm$  mesons. It is well known that, at very low energy, there should be a cut-off which correspond to energy scale of chiral symmetry spontaneous broken,  $\Lambda_{\text{CSSB}}$ . In presence of spin-1 mesons, another cut-off should be introduced with energy increasing. Unfortunately, there are not any practical methods helping us to understand properties of this cut-off from QCD. It is only expected that, due to requirement of consistence, this cut-off should be larger than  $\Lambda_{\text{CSSB}}$  and all spin-1 meson masses. In sect. 3, we will fit values of these two cut-offs phenomenologically and show that they are consistent indeed.

As soon as we start studying the next to leading order of  $1/N_c$  expansion, there are two contributions contained the same as at leading order. One contribution is from meson loops and another contribution is from quark-gluon coupling. It has to be recognized that it is too hard to systematically study quark-gluon coupling at the next to leading order of  $1/N_c$  expansion. The reason is that we will face many uncertain ingredients and very tedious calculation. In fact, even though at leading order of  $1/N_c$  expansion, it is impossible to completely investigate the contribution from quark-gluon coupling, since theoretically, the numbers of these kinds of feynman diagrams are infinite(see detail in sect. 4). Fortunately, the dominant contribution can be captured here. The phenomenological results show that the quark-gluon coupling is small even at leading order. We can expect that it is legitimate to ignore the next to leading order contribution of quark-gluon coupling.<sup>3</sup> Due to these reasons, the effects of quark-gluon coupling will be omitted at the next to leading order of  $1/N_c$ . The main content of the present paper is to study effects of one-loop of pseudoscalar, vector and axial-vector mesons systematically.

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<sup>3</sup>In this paper, the contribution of quark-gluon coupling denotes the exchange effects of gluon in internal of quark loop. In this case gluon is *soft*(see discussion in sect. 8). Here we ignore another kind of gluon effect which is exchange of gluon between two quark loops, since study on this effect is beyond the method provided by ref.[15]. In this case gluon can be *hard*. It is well known that this effect is suppressed by  $1/N_c$  expansion and OZI rule.

Obviously, the calculation on meson one-loop effects is beyond one of ChPT in which only pseudoscalar meson one-loop graphs are involved. So that it will help us to understand the dynamics with energy scale lying between perturbative QCD and ChPT more accurately. Especially, many processes suppressed by  $1/N_c$  will become calculable explicitly. However, although in principle there are no problems on calculation of meson one-loop graphs, practical calculation is very tedious due to complicated dynamics structure of effective lagrangian and due to various high-order divergences coming from propagators of spin-1 mesons (see sect. 6). Hence we need to search a method to simplify our calculation. Noting that we work in the framework of a non-renormalizable truncated field theory, we have to introduce some cut-offs to regularize divergences from meson loops. These cut-offs can be interpreted as meson-interaction scale in the “effective vertices” generated by meson loops. Therefore, the effects of meson loops describe “very long-distance” correction comparing with one of quark loops. Contribution of the former should be much smaller than the latter. In the other word, the cut-offs from meson loops should be much smaller than one from quark loops, e.g.,  $\Lambda_{\text{CSSB}}$ . It indicates that momentum transfer in meson loops is small. If these cut-offs is smaller than vector meson masses, the dominant contributions will come from logarithmic divergences and all high order divergences can be omitted. Sequentially, the complicated calculation will be simplified. In sect. 6, this point will be examined explicitly.

Although we still do not know how to obtain a low energy effective theory from QCD directly, relationship between ChQM and ChPT is straightforward. At very low energy, when the freedoms of spin-1 mesons are “freezed” (i.e., integrating out these freedoms by path-integral), ChQM should return to ChPT. It is well known that ChPT is a rigorous consequence of the symmetry pattern in QCD and its spontaneous breaking. So that it is necessary to examine low energy limit of ChQM and check whether it agrees with ChPT or not. In this article, we will calculate ten coupling constants of ChPT at  $O(p^4)$  from ChQM and show that they agree with ChPT well.

The paper is organized as follows. In sect. 2, we give a brief review on ChQM and extend ChQM to include spin-1 meson resonances. In sect. 3, we derive effective lagrangian of mesons generated by quark loops. In sect. 4, the correction of quark-gluon coupling is studied. We further compare ChQM and QCD sum rules and determine the value of gluon condensate obtained through integrating over quark and gluon fields. General formulas for meson one-loop contribution are in sect. 5 and systematical study on meson one-loop graphs is in sect. 6. In sect 7, we discuss low energy limit of ChQM and some phenomenological results. Sect. 8 is devoted to summary and discussion.

## 2 $SU(3)_L \times SU(3)_R$ Chiral Quark Model of Mesons

The QCD lagrangian with three flavor quark  $\bar{\psi} = (\bar{u}, \bar{d}, \bar{s})$  is,

$$\begin{aligned}\mathcal{L}_{QCD}(x) &= \mathcal{L}_{QCD}^0 + \bar{\psi}(\gamma \cdot v + \gamma \cdot a \gamma_5)\psi - \bar{\psi}(s - i\gamma_5 p)\psi, \\ \mathcal{L}_{QCD}^0 &= \bar{\psi}(i\gamma \cdot \partial + \gamma \cdot G)\psi - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu}, \quad (a = 1, 2, \dots, 8),\end{aligned}\tag{2.1}$$

where the  $3 \times 3$  gluon field matrix is given by

$$G_\mu = g_s \frac{\lambda^a}{2} G_\mu^a(x),\tag{2.2}$$

and

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f_{abc} G_\mu^b G_\nu^c \quad (2.3)$$

is the gluon field strength tensor and  $g_s$  is the color coupling constant ( $\alpha_s = g_s^2/4\pi$ ). The fields  $v_\mu, a_\mu$  and  $p$  are  $3 \times 3$  matrices in flavor space and denote respectively vector, axial-vector and pseudoscalar external fields.  $s = \mathcal{M} + s_{\text{ext}}$ , where  $s_{\text{ext}}$  is scalar external fields and  $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$  is quark mass matrix with three flavor.

The introduction of external fields  $v_\mu$  and  $a_\mu$  allows for the global symmetry of the lagrangian to be invariant under local  $SU(3)_L \times SU(3)_R$ , i.e., with  $g_L, g_R \in SU(3)_L \times SU(3)_R$ , the explicit transformations of the different fields are

$$\begin{aligned} \psi(x) &\rightarrow g_R(x) \frac{1}{2}(1 + \gamma_5) \psi(x) + g_L(x) \frac{1}{2}(1 - \gamma_5) \psi(x), \\ l_\mu &\equiv v_\mu - a_\mu \rightarrow g_L(x) l_\mu g_L^\dagger(x) + i g_L(x) \partial_\mu g_L^\dagger(x), \\ r_\mu &\equiv v_\mu + a_\mu \rightarrow g_R(x) r_\mu g_R^\dagger(x) + i g_R(x) \partial_\mu g_R^\dagger(x), \\ s + ip &\rightarrow g_R(x) (s + ip) g_L^\dagger(x). \end{aligned} \quad (2.4)$$

What can be physical observable is the generating functional of Green's function of vector, axial-vector, scalar and pseudoscalar external fields,  $v, a, s, p$ . The generating functional can be calculated in the path-integral formalism as,

$$e^{iW_{QCD}[v, a, s, p]} = \frac{1}{N} \int \mathcal{D}G_\mu \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\{i \int d^4x \mathcal{L}_{QCD}(G, \psi, \bar{\psi}; v, a, s, p)\}. \quad (2.5)$$

At the low energies the coupling constant becomes strong and perturbative QCD can no longer be done so that we need some low energy effective models (quark model, pole model, Skyrme model,...) to approach low energy behaviors of QCD.

The simplest pattern of chiral quark model is described by the following chiral quark Lagrangian (M-G model) with three flavor massless quarks [2]

$$\mathcal{L}_{M-G} = \bar{\psi}(x) (i\gamma \cdot \nabla - m u(x)) \psi(x), \quad (2.6)$$

where

$$\begin{aligned} \nabla_\mu &= \partial_\mu - i v_\mu - i a_\mu \gamma_5 \\ u(x) &= \frac{1}{2}(1 - \gamma_5) U(\Phi) + \frac{1}{2}(1 + \gamma_5) U^\dagger(\Phi), \\ U(\Phi) &= \exp\{2i\lambda^a \Phi^a(x)\}, \end{aligned} \quad (2.7)$$

and  $\lambda_a$  are Gell-Mann matrices of  $SU(3)_{\text{flavor}}$ ,  $\Phi_a$  are fields of pseudoscalar mesons octet of  $SU(3)$ . Under local  $SU(3)_L \times SU(3)_R$ , the explicit transformations of pseudoscalar fields is

$$U(\Phi) \longrightarrow g_R(x) U(\Phi) g_L(x) \quad (2.8)$$

We like to make several remarks about M-G model: 1)  $\mathcal{L}_{M-G}$  is invariant under global chiral transformations. 2) Making a chiral rotation of quark fields,  $\chi_L = \xi(\Phi) \psi_L$ ,  $\chi_R = \xi^\dagger(\Phi) \psi_R$ , the second term of  $\mathcal{L}_{M-G}$  reduces to a mass term for the dressed quarks  $\chi$  (i.e., constituent quarks),

so that the parameter  $m$  can then be interpreted as a constituent quark mass. 3) In  $\mathcal{L}_{M-G}$ , the operator  $\bar{\psi}\psi$  acquires a vacuum expectation value. Therefore this is an effective way to generate the order parameter due to CSSB[15, 16]. 4) Due to the smallness of effective gluon couplings, the contributions of gluons are perturbative correction in  $\mathcal{L}_{M-G}$ . 5) Note that there are no kinetic terms for  $\Phi_a$  in original  $\mathcal{L}_{M-G}$ . The kinetic terms of pseudoscalar fields are reduced by quantum fluctuation effects due to quark-loops[3]-[6]. Therefore there is no so called double counting problem.  $\Phi_a$  are actually the composite fields of quarks. 6) A very low energy strong interaction theory involving pseudoscalar mesons only can be derived via integrating over the freedom of quarks.

By means of M-G model, the quark mass-independent low energy coupling constants have been derived in Refs.[7, 15]. It is remarkable that the predictions of this simple model are in agreement with the phenomenological values of  $L_i$  in ChPT. This means the low energy limit M-G model is compatible with ChPT in chiral limit. In the baryon physics, the skyrmion calculations show also that the predictions from M-G model are reasonable[3, 17, 18]. In Ref.[15] the leading effective gluonic corrections to  $L_i$  are extensively discussed. Furthermore, in Ref.[19] the authors proved that the interaction in  $\mathcal{L}_{M-G}$  is equivalent to the mean-field approximation of the Nambu-Jona-Lasinio model[4]. The facts indicate that M-G model is sound as a base for the investigations of low energy meson physics.

In this paper, we like to extend M-G model to include spin-1 meson resonances. From the viewpoint of chiral symmetry only, vector and axial-vector mesons do not have any special status compared to pseudoscalar mesons. As pointed out in Ref.[11], all spin-1 meson resonances carry non-linear realizations of global chiral group  $G = SU(3)_L \times SU(3)_R$  depending on their transformation properties under the subgroup  $H = SU(3)_V$ . A non-linear realization of spontaneously broken chiral symmetry is defined in [20] by specifying the action of G on elements  $\xi(\Phi)$  of the coset space G/H:

$$\xi(\Phi) \rightarrow g_R \xi(\Phi) h^\dagger(\Phi) = h(\Phi) \xi(\Phi) g_L^\dagger. \quad (2.9)$$

Explicit form of  $\xi(\Phi)$  is usually taken

$$\xi(\Phi) = \exp \{i\lambda^a \Phi^a(x)\}, \quad U(\Phi) = \xi^2(\Phi).$$

The compensating  $SU(3)_V$  transformation  $h(\Phi)$  defined by Eq.( 2.9) is the wanted ingredient for a non-linear realization of G. In practice, we shall only be interested in spin-1 meson resonances transforming as octets and singlets under  $SU(3)_V$ . Denoting the multiplets generically be  $O_\mu$ (octet) and  $O_{1\mu}$ (singlet), the non-linear realization of G is given by

$$O_\mu \rightarrow h(\Phi) O_\mu h^\dagger(\Phi), \quad O_{1\mu} \rightarrow O_{1\mu}. \quad (2.10)$$

More convenience, due to OZI rule, the vector and axial-vector octets and singlets are combined into a single “nonet” matrix

$$N_\mu = O_\mu + \frac{I}{\sqrt{3}} O_{1\mu}, \quad N_\mu = V_\mu, A_\mu,$$

where

$$V_\mu(x) = \lambda \cdot \mathbf{V}_\mu = \sqrt{2} \begin{pmatrix} \frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & \bar{K}_\mu^{*0} & \phi_\mu \end{pmatrix}, \quad (2.11)$$

$$A_\mu(x) = \lambda \cdot \mathbf{A}_\mu = \sqrt{2} \begin{pmatrix} \frac{a_{1\mu}^0}{\sqrt{2}} + \frac{f_\mu}{\sqrt{2}} & a_{1\mu}^+ & K_{1\mu}^+ \\ a_{1\mu}^- & -\frac{a_{1\mu}^0}{\sqrt{2}} + \frac{f_\mu}{\sqrt{2}} & K_{1\mu}^0 \\ K_{1\mu}^- & \bar{K}_{1\mu}^0 & f_{1\mu} \end{pmatrix}. \quad (2.12)$$

From viewpoint of phenomenology, we can find in Eq.( 2.6) that photon field and  $W$ ,  $Z$ -fields enter dynamics of hadrons through the coupling to quarks via covariant derivative  $\nabla_\mu$ . In addition, it is well known that in the electromagnetic and weak interactions of mesons the vector ( $V$ ) and axial-vector mesons ( $A$ ) play essential role through VMD (Vector Meson Dominate) and AVMD (Axial-Vector Meson Dominate)[21], and enjoy considerable phenomenological support. This indicates that it is available to substitute new affine connection  $L_\mu + l_\mu$  and  $R_\mu + r_\mu$  for the old  $l_\mu$  and  $r_\mu$  in Eq.( 2.6), where the auxiliary fields  $L_\mu = \xi^\dagger(V_\mu - A_\mu)\xi$  and  $R_\mu = \xi(V_\mu + A_\mu)\xi^\dagger$ . Therefore, we can naturally extend M-G model to be one including the lowest meson resonances with spin-1 via minimum coupling principle,

$$\mathcal{L}_\chi = \bar{\psi}(x)(i\gamma \cdot D - mu(x))\psi(x) + \frac{1}{2}m_1^2(V_\mu^a V^{\mu a}) + \frac{1}{2}m_2^2(A_\mu^a A^{\mu a}), \quad (2.13)$$

where <sup>4</sup>

$$\nabla_\mu \longrightarrow D_\mu \equiv \partial_\mu - i\frac{1-\gamma_5}{2}(L_\mu + l_\mu) - i\frac{1+\gamma_5}{2}(R_\mu + r_\mu) \quad (2.14)$$

The transformation( 2.10) leads to that the mass type terms of vector and axial-vector composite fields are allowed to be added in the lagrangian  $\mathcal{L}_\chi$ . The  $L'_\mu = L_\mu + l_\mu$  and  $R'_\mu = R_\mu + r_\mu$  transform nonlinearly as gauge bosons under the chiral group. <sup>5</sup> Making the chiral rotation from current quark fields to constituent quark fields,  $\chi_L = \xi(\Phi)\psi_L$ ,  $\chi_R = \xi^+(\Phi)\psi_R$ , the spin-1 meson fields will couple to constituent quark fields directly. Therefore, spin-1 meson resonances in this framework are the composite fields of constituent quark fields instead of the one of current quark fields in Ref.[5, 6].

Finally, light quark mass-dependent term can be included in ChQM in terms of standard form

$$\bar{\psi}(s - i\gamma_5 p)\psi. \quad (2.15)$$

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<sup>4</sup>The vector meson field  $V_\mu$  and axial-vector meson field  $A_\mu$  in  $D_\mu$  can combine with coupling constants  $g_V$  and  $g_A$  respectively. However, these constants will be absorbed in mass term of spin-1 mesons via the field rescaling  $V_\mu \rightarrow \frac{1}{g_V}V_\mu$  and  $A_\mu \rightarrow \frac{1}{g_A}A_\mu$ .

<sup>5</sup>The  $U(1)_A$  anomaly effects are absent in this present paper, which will be studied in elsewhere.

Then general ChQM lagrangian is written

$$\begin{aligned}\mathcal{L}_\chi = & \mathcal{L}_{QCD}^0 + \bar{\psi}_L \gamma^\mu (L_\mu + l_\mu) \psi_L + \bar{\psi}_R \gamma^\mu (R_\mu + r_\mu) \psi_R \\ & - m \bar{\psi} u(x) \psi - \bar{\psi} (s - i\gamma_5 p) \psi + \frac{1}{2} m_1^2 V_\mu^a V^{a\mu} + \frac{1}{2} m_2^2 A_\mu^a A^{a\mu},\end{aligned}\quad (2.16)$$

where

$$\psi_L = \frac{1 - \gamma_5}{2} \psi, \quad \psi_R = \frac{1 + \gamma_5}{2} \psi. \quad (2.17)$$

Since the composite pseudoscalar, vector and axial-vector meson fields are treated as background fields, there are no kinetic terms for them in lagrangian (2.16). They will be generated by quark-loop effects.

Our purpose is to use chiral quark model to approach low energy behavior of QCD. In other words, we replace the generating functional of Green's function of external fields in QCD (2.5) by one of chiral quark model,

$$e^{i\bar{W}[V,A,U;v,a,s,p]} = \frac{1}{N} \int \mathcal{D}G \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\{i \int d^4x \mathcal{L}_\chi\}. \quad (2.18)$$

The relationship between generating functional  $W_{QCD}$  and  $\bar{W}$  is as follows: As energy scale  $\mu$  is higher than chiral symmetry spontaneous breaking scale  $\Lambda_{CSSB}$ , the composite mesonic fields disappear. Then we have

$$W_{QCD}[v, a, s, p] = \bar{W}[0, 0, 1; v, a, s, p]|_{m=0}. \quad (2.19)$$

As  $\mu < \Lambda_{CSSB}$  mesonic fields will appear as dynamical freedom in the theory. Then we obtain

$$e^{iW_{QCD}[v,a,s,p]} = \frac{1}{N} \int \mathcal{D}U \mathcal{D}V_\mu \mathcal{D}A_\mu e^{i\bar{W}[V,A,U;v,a,s,p]} \quad (2.20)$$

### 3 Integrating Out Quark Fields

#### 3.1 Effective lagrangian from quark loops

Our goal is to derive the effective lagrangian of mesons up to the next to leading order of  $1/N_c$  expansion. It is expressed as follows

$$\mathcal{L}_{eff} = \mathcal{L}_{eff}^{(0)} + \mathcal{L}_{eff}^{(g)} + \mathcal{L}_{eff}^{(l)} \quad (3.1)$$

where  $\mathcal{L}_{eff}^{(0)}$  comes from quark loops,  $\mathcal{L}_{eff}^{(g)}$  and  $\mathcal{L}_{eff}^{(l)}$  are contributed by quark-gluon coupling and one-loop of mesons respectively. According to the large  $N_c$  expansion argument [14],  $\mathcal{L}_{eff}^{(0)} \sim \mathcal{O}(N_c)$  is the leading order of  $\mathcal{L}_{eff}$ , and  $\mathcal{L}_{eff}^{(l)} \sim \mathcal{O}(1)$  is next leading order of  $\mathcal{L}_{eff}$ . In addition, according to discussion in sect. 1, we treat  $\mathcal{L}_{eff}^{(g)}$  as perturbation although  $\mathcal{L}_{eff} \sim \mathcal{O}(N_c)$  is also the leading order. In this section, we derive  $\mathcal{L}_{eff}^{(0)}$  in terms of integrating out the quark fields in lagrangian (2.16).



A review for chiral gauge theory is in [22]. The effective lagrangian of mesons in ChQM can be obtained in Euclidian space by means of integrating over degrees of freedom of fermions

$$\exp\{-\int d^4x \mathcal{L}_{eff}^{(0)}\} = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \exp\{-\int d^4x \mathcal{L}_\chi\}. \quad (3.2)$$

Then we have

$$\mathcal{L}_{eff}^{(0)} = -\ln \det \mathcal{D}, \quad (3.3)$$

with

$$\mathcal{D} = \gamma^\mu (\partial_\mu - i \frac{1-\gamma_5}{2} (L_\mu + l_\mu) - i \frac{1+\gamma_5}{2} (R_\mu + r_\mu)) + mu + (s - ip\gamma_5). \quad (3.4)$$

The gluon fields are absent in the operator  $\mathcal{D}$  since the quark-gluon coupling is treated as perturbation. The effective lagrangian is separated into two parts

$$\begin{aligned} \mathcal{L}_{eff}^{(0)} &= \mathcal{L}_{eff}^{Re} + \mathcal{L}_{eff}^{Im} \\ \mathcal{L}_{eff}^{Re} &= -\frac{1}{2} \ln \det (\mathcal{D} \mathcal{D}^\dagger), \quad \mathcal{L}_{eff}^{Im} = -\frac{1}{2} \ln \det [(\mathcal{D}^\dagger)^{-1} \mathcal{D}] \end{aligned} \quad (3.5)$$

where

$$\mathcal{D}^\dagger = \gamma_5 \hat{\mathcal{D}} \gamma_5, \quad (3.6)$$

and  $\hat{B} = \frac{1}{2}(1 + \gamma_5)B_L + \frac{1}{2}(1 - \gamma_5)B_R$  for arbitrarily operator  $B = \frac{1}{2}(1 - \gamma_5)B_L + \frac{1}{2}(1 + \gamma_5)B_R$ . The effective lagrangian  $\mathcal{L}_{eff}^{Re}$  describes the physical processes with normal parity and  $\mathcal{L}_{eff}^{Im}$  describes the processes with anomalous parity. In the present paper we focus our attention on  $\mathcal{L}_{eff}^{Re}$ . The discussion of  $\mathcal{L}_{eff}^{Im}$  can be found in Refs.[22]. In terms of Schwinger's proper time method [23],  $\mathcal{L}_{eff}^{Re}$  is written as

$$\mathcal{L}_{eff}^{Re} = -\frac{1}{2\delta(0)} \int d^4x \frac{d^4p}{(2\pi)^4} Tr \int_0^\infty \frac{d\tau}{\tau} (e^{-\tau \mathcal{D}'^\dagger \mathcal{D}'} - e^{-\tau \Delta_0}) \delta^4(x-y)|_{y \rightarrow x} \quad (3.7)$$

with

$$\begin{aligned} \mathcal{D}' &= \mathcal{D} - i\gamma \cdot p, & \mathcal{D}'^\dagger &= \mathcal{D}^\dagger + i\gamma \cdot p, \\ \Delta_0 &= p^2 + M^2. \end{aligned} \quad (3.8)$$

where  $M$  is an arbitrary parameter with dimension of mass. The Seeley-DeWitt coefficients or heat kernel method have been used to evaluate the expansion series of Eq.( 3.8). In this paper we will use dimensional regularization. After completing the integration over  $\tau$ , the lagrangian  $\mathcal{L}_{eff}^{Re}$  reads

$$\mathcal{L}_{eff}^{Re} = -\frac{\mu^\epsilon}{2\delta(0)} \int d^Dx \frac{d^Dp}{(2\pi)^D} \sum_{i=1}^\infty \frac{1}{n\Delta_0^n} Tr (\mathcal{D}'^\dagger \mathcal{D}' - \Delta_0)^n \delta^D(x-y)|_{y \rightarrow x}, \quad (3.9)$$

where trace is taken over the color, flavor and Lorentz space. This effective lagrangian can be expanded in powers of derivatives,<sup>6</sup>

$$\mathcal{L}_{Re} = \mathcal{L}_2^{(0)} + \mathcal{L}_4^{(0)} + \dots \quad (3.10)$$

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<sup>6</sup>Here we need to distinguish expansion in powers of derivatives from low energy expansion in ChPT. It will be discussed in sect. 3.2.

In Minkowski space, effective lagrangian with two derivatives reads

$$\begin{aligned}\mathcal{L}_2^{(0)} &= \lambda(\mu)m^2 < D_\mu U D^\mu U^\dagger > + \frac{N_c}{(4\pi)^{D/2}} \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \Gamma(1 - \frac{D}{2}) \frac{m^3}{B_0} < \chi U^\dagger + \chi^\dagger U > \\ &\quad + \frac{1}{4} m_1^2 < V_\mu V^\mu > + \frac{1}{4} m_2^2 < A_\mu A^\mu >\end{aligned}\quad (3.11)$$

where “ $< \dots >$ ” denotes trace in flavor space and

$$\begin{aligned}\lambda(\mu) &= \frac{N_c}{(4\pi)^{D/2}} \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \\ \chi &= 2B_0(s + ip) \\ D_\mu U &= \nabla_\mu U - 2i\xi A_\mu \xi, \\ D_\mu U^\dagger &= \nabla_\mu U^\dagger + 2i\xi^\dagger A_\mu \xi^\dagger, \\ \nabla_\mu U &= \partial_\mu U + iU l_\mu - ir_\mu U, \\ \nabla_\mu U^\dagger &= \partial_\mu U^\dagger + iU^\dagger l_\mu - ir_\mu U^\dagger.\end{aligned}\quad (3.12)$$

In lagrangian ( 3.11) the axial-vector fields  $A_\mu$  mixes with pseudoscalar fields,  $\partial_\mu \Phi$ . This mixing should be diagonalized via field shift

$$A_\mu \longrightarrow A_\mu - ic\Delta_\mu, \quad c = \frac{\lambda(\mu)m^2}{\lambda(\mu)m^2 + m_2^2}, \quad (3.13)$$

where

$$\Delta_\mu = \frac{1}{2} \{ \xi^\dagger (\partial_\mu - ir_\mu) \xi - \xi (\partial_\mu - il_\mu) \xi^\dagger \} = \frac{1}{2} \xi^\dagger \nabla_\mu U \xi^\dagger = -\frac{1}{2} \xi \nabla_\mu U^\dagger \xi. \quad (3.14)$$

Then effective lagrangian with two derivatives reads

$$\mathcal{L}_2^{(0)} = \frac{f_0^2}{16} < \nabla_\mu U \nabla^\mu U^\dagger + \xi U^\dagger + \xi^\dagger U > + \frac{1}{4} m_1^2 < V_\mu V^\mu > + \frac{1}{4} \bar{m}_2^2 < A_\mu A^\mu >. \quad (3.15)$$

In above lagrangian we have defined the constants  $f_0$ ,  $B_0$  and  $\bar{m}_2$  to absorb divergences from quark loop integral

$$\begin{aligned}\frac{f_0^2}{16} &= \lambda(\mu)m^2(1 - c), \\ \frac{f_0^2}{16} B_0 &= \frac{N_c}{(4\pi)^{D/2}} \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \Gamma(1 - \frac{D}{2}) m^3, \\ \bar{m}_2^2 &= 16\lambda(\mu)m^2 + m_2^2.\end{aligned}\quad (3.16)$$

It should be noted that the field shift( 3.13) is different from that one in Refs.[6, 13]. This field shift is convenient to keep gauge symmetry explicitly. Moreover, it make that there are no spin-1 mesons coupling to pseudoscalar fields in  $\mathcal{L}_2^{(0)}$ . We will show in sect. 6 that this result is useful in calculation on meson loops.

The lagrangian ( 3.11), ( 3.15) and Eq.( 3.16) yield two equivalent equations of motion of pseudoscalar mesons as follows

$$\begin{aligned} D_\mu(UD^\mu U^\dagger) + \frac{1-c}{2}(\chi U^\dagger - U\chi^\dagger) &= 0, \\ \nabla_\mu(U\nabla^\mu U^\dagger) + \frac{1}{2}(\chi U^\dagger - U\chi^\dagger) &= 0. \end{aligned} \quad (3.17)$$

The first equation is in presence of axial-vector mesons and the second one is in absence of axial-vector mesons. All pseudoscalar meson fields satisfy these equations.

Since the non-linear realization of G on the spin-1 meson fields  $\mathcal{O}$  in expression ( 2.10) is local we are led to define a covariant derivative

$$d_\mu \mathcal{O} = \partial_\mu \mathcal{O} + [\Gamma_\mu, \mathcal{O}], \quad (3.18)$$

with

$$\Gamma_\mu = \frac{1}{2}\{\xi^\dagger(\partial_\mu - ir_\mu)\xi + \xi(\partial_\mu - il_\mu)\xi^\dagger\}, \quad (3.19)$$

ensuring the proper transformation

$$d_\mu \mathcal{O} \longrightarrow h(\Phi)d_\mu \mathcal{O} h^\dagger(\Phi).$$

Without external fields,  $\Gamma_\mu$  is the usual natural connection on coset space.

Then from Eq.( 3.9) and due to equation of motion( 3.17), effective lagrangian with four derivatives can be obtained in Minkowski space as follows

$$\begin{aligned} \mathcal{L}_4^{(0)} &= -\frac{\lambda_r(\mu)}{16} \langle L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} \rangle - \frac{\gamma}{6} \langle L_{\mu\nu} U^\dagger R^{\mu\nu} U \rangle \\ &\quad - \frac{i\gamma}{3} \langle D_\mu U D_\nu U^\dagger R^{\mu\nu} + D_\mu U^\dagger D_\nu U L^{\mu\nu} \rangle + \frac{\gamma}{12} \langle D_\mu U D_\nu U^\dagger D^\mu U D^\nu U^\dagger \rangle \\ &\quad + \theta_1 \langle D_\mu U D^\mu U^\dagger (\chi U^\dagger + \chi^\dagger U) \rangle + \theta_2 \langle \chi U^\dagger \chi U^\dagger + \chi^\dagger U \chi^\dagger U \rangle \end{aligned} \quad (3.20)$$

where

$$\begin{aligned} \gamma &= \frac{N_c}{(4\pi)^2}, & \lambda_r(\mu) &= \frac{8}{3}\lambda(\mu) - \frac{4}{3}\gamma, \\ \theta_1 &= (\lambda(\mu) - \gamma) \frac{m}{2B_0}, \\ \theta_2 &= \frac{\lambda(\mu)m}{4B_0} \left(1 - c - \frac{m}{B_0}\right) - \frac{\gamma}{24} (1 - c)^2. \end{aligned} \quad (3.21)$$

Due to field shift ( 3.13), in  $\mathcal{L}_4^{(0)}$  we have defined

$$\begin{aligned} L_{\mu\nu} &= (1 - \frac{c}{2})F_{\mu\nu}^L + \frac{c}{2}F_{\mu\nu}^R + \xi^\dagger(V_{\mu\nu} - A_{\mu\nu})\xi - 2ic(1 - \frac{c}{2})\xi^\dagger[\Delta_\mu, \Delta_\nu]\xi \\ &\quad - (1 - c)\xi^\dagger([\Delta_\mu, V_\nu - A_\nu] - [\Delta_\nu, V_\mu - A_\mu])\xi, \\ R_{\mu\nu} &= (1 - \frac{c}{2})F_{\mu\nu}^R + \frac{c}{2}F_{\mu\nu}^L + \xi(V_{\mu\nu} + A_{\mu\nu})\xi^\dagger - 2ic(1 - \frac{c}{2})\xi[\Delta_\mu, \Delta_\nu]\xi^\dagger \\ &\quad + (1 - c)\xi([\Delta_\mu, V_\nu + A_\nu] - [\Delta_\nu, V_\mu + A_\mu])\xi^\dagger, \\ D_\mu U &= (1 - c)\nabla_\mu U - 2i\xi A_\mu \xi, \\ D_\mu U^\dagger &= (1 - c)\nabla_\mu U^\dagger + 2\xi^\dagger A_\mu \xi^\dagger, \end{aligned} \quad (3.22)$$

with

$$\begin{aligned}
F_{\mu\nu}^{R,L} &= \partial_\mu(v_\nu \pm a_\nu) - \partial_\nu(v_\mu \pm a_\mu) - i[v_\mu \pm a_\mu, v_\nu \pm a_\nu]. \\
V_{\mu\nu} &= d_\mu V_\nu - d_\nu V_\mu - i[V_\mu, V_\nu] - i[A_\mu, A_\nu] \\
A_{\mu\nu} &= d_\mu A_\nu - d_\nu A_\mu - i[A_\mu, V_\nu] - i[V_\mu, A_\nu].
\end{aligned} \tag{3.23}$$

### 3.2 Physical effective lagrangian and beyond low energy expansion

In general, the effective lagrangian(it need not to be the leading order of  $1/N_c$ ) can be rewritten as follows

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_2^\phi + \mathcal{L}_4^\phi + \mathcal{L}_{\text{kin}}^{V,A} + \mathcal{L}_1^{V,A} + \dots, \tag{3.24}$$

where  $\mathcal{L}^\phi$  denotes effective lagrangian describing interaction of pseudoscalar mesons in very low energy but without spin-1 meson resonances(where  $\phi$  denotes pseudoscalar fields),  $\mathcal{L}_{\text{kin}}^{V,A}$  is kinetic terms of spin-1 mesons and  $\mathcal{L}_1^{V,A}$  denotes effective lagrangian describing spin-1 mesons coupling to pseudoscalar meson. Since there is no interaction of spin-1 mesons in  $\mathcal{L}_2$ , all terms in  $\mathcal{L}_1^{V,A}$  are with four derivatives. In very low energy, equation of motion  $\delta\mathcal{L}/\delta\mathcal{O} = 0$ , ( $\mathcal{O} = V, A$ ) yields classic solution of spin-1 mesons as follows

$$O_\mu = \frac{1}{m_\phi^2} \times O(p^3) \text{ terms,}$$

where  $p$  is momentum of pseudoscalar mesons in very low energy. In low energy limit, degrees of freedom of spin-1 meson resonances disappear and their dynamics is replaced by pseudoscalar fields, hence in this energy region  $\mathcal{L}^{V,A}$  is  $O(p^6)$ . It means that, up to  $O(p^4)$ ,  $\mathcal{L}_2^\phi + \mathcal{L}_4^\phi$  is just low energy limit of ChQM. If there are processes in which all external lines are pseudoscalar mesons, spin-1 meson fields do not appear in internal lines of tree diagrams of these processes. It seems to be different from the results in some Refs.[11, 7] in which chiral coupling in very low energy receive large contribution from spin-1 meson exchange. However, we like to point out that, this difference is caused by definition of physical meson fields and does not change physical results. In this paper, the physical meson fields are defined by expression ( 2.7), ( 2.10) and field shift ( 3.13).

There are divergences from quark loops in  $\mathcal{L}^{(0)\phi}$  and  $\mathcal{L}^{(0)V,A}$ (recalling that superscript “(0)” denotes effective lagrangian generated by quark loops). In sect. 3.1, divergences in  $\mathcal{L}_2^{(0)}$  have been absorbed by  $f_0$ ,  $B_0$  and axial-vector mass-dependent parameter  $\bar{m}_2$ . Here we need to define two constants  $g_\phi^2 = \frac{8}{3}\lambda(\mu_\phi)$  and  $g^2 = \frac{8}{3}\lambda(\mu_V)$  to absorb logarithmic divergences in  $\mathcal{L}_4^{(0)\phi}$  and  $\mathcal{L}^{(0)V,A}$  respectively since scale factor  $\mu$  of dimensional regularization is arbitrary. Equivalently, it means that two cut-offs are needed here. One corresponds to very low energy and regularize logarithmic divergences in  $\mathcal{L}_4^{(0)\phi}$ , Another corresponds to energy scale of vector meson masses and regularize logarithmic divergences in  $\mathcal{L}^{(0)V,A}$ . It has been mentioned that these divergences are caused by “point particles” approximation of composite mesons in lagrangian ( 2.16). Therefore, the these cut-offs can be treated as description of interaction scale of quark pairs in mesons. Naturally, these interaction scales are different for spin zero and spin-1 mesons. It was also shown by some model. For instance, effective potential model[24] showd that there are obvious difference for mesons with different spin content. Thus  $g_\phi \neq g$  is a nature result.

The explicit form of  $\mathcal{L}_2^{(0)\phi}$  has been shown in Eq.( 3.15). In general, chiral symmetry requires  $\mathcal{L}_4^\phi$  taking the form as follows[9]

$$\begin{aligned}\mathcal{L}_4^\phi = & L_1 < \nabla_\mu U \nabla^\mu U^\dagger >^2 + L_2 < \nabla_\mu U \nabla_\nu U^\dagger \nabla_\mu U \nabla_\nu U^\dagger > + L_3 < \nabla_\mu U \nabla^\mu U^\dagger \nabla_\nu U \nabla^\nu U^\dagger > \\ & + L_4 < \nabla_\mu U \nabla^\mu U^\dagger > < \chi U^\dagger + \chi^\dagger U > + L_5 < \nabla_\mu U \nabla^\mu U^\dagger (\chi U^\dagger + \chi^\dagger U) > \\ & + L_6 < \chi U^\dagger + \chi^\dagger U >^2 + L_7 < \chi U^\dagger - \chi^\dagger U >^2 + L_8 < \chi U^\dagger \chi U^\dagger + \chi^\dagger U \chi^\dagger U > \\ & - i L_9 < \nabla_\mu U \nabla_\nu U^\dagger F^{R\mu\nu} + \nabla_\mu U^\dagger \nabla_\nu U F^{L\mu\nu} > + L_{10} < F_{\mu\nu}^L U^\dagger F^{R\mu\nu} U >, \quad (3.25)\end{aligned}$$

where  $L_i (i = 1, 2, \dots, 10)$  are ten real constants which determine dynamics of pseudoscalar meson interaction at very low energy, together with  $f_0$  and  $B_0$ . From Eq.( 3.20, these low energy coupling constants read

$$\begin{aligned}g_\phi^2 &= \frac{8}{3} \frac{N_c}{(4\pi)^{D/2}} \left( \frac{\mu_\phi^2}{m^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}), \\ L_1^{(0)} &= \frac{1}{2} L_2^{(0)} = \frac{g_\phi^2}{32} c^2 (1 - \frac{c}{2})^2 + \frac{\gamma}{6} c (1 - \frac{c}{2}) (1 - c)^2 + \frac{\gamma}{24} (1 - c)^4, \\ L_3^{(0)} &= -\frac{3}{16} g_\phi^2 c^2 (1 - \frac{c}{2})^2 - \gamma c (1 - \frac{c}{2}) (1 - c)^2 - \frac{\gamma}{6} (1 - c)^4, \\ L_5^{(0)} &= (\frac{3}{8} g_\phi^2 - \gamma) \frac{m}{2B_0} (1 - c)^2, \\ L_8^{(0)} &= \frac{3g_\phi^2 m}{32B_0} (1 - c - \frac{m}{B_0}) - \frac{\gamma}{24} (1 - c)^2, \\ L_9^{(0)} &= \frac{g_\phi^2}{8} c (1 - \frac{c}{2}) + \frac{\gamma}{3} (1 - c)^2, \\ L_{10}^{(0)} &= -\frac{g_\phi^2}{8} c (1 - \frac{c}{2}) - \frac{\gamma}{6} (1 - c)^2. \\ L_4^{(0)} &= L_6^{(0)} = L_7^{(0)} = 0.\end{aligned} \quad (3.26)$$

Input experimental data  $L_9 = (6.9 \pm 0.7)^{-3}$  we can obtain  $g_\phi = 0.32 \pm 0.02$ <sup>7</sup> (the value of  $c = 0.44$  will be fitted in the following).

Since all meson fields in  $\mathcal{L}^\phi$  are pseudoscalars, the low energy expansion of  $\mathcal{L}^\phi$  is well-defined. However, it is known that the momentum expansion in  $\mathcal{L}^{V,A}$  is very unclear. Then how do we know that our calculation on physics in this energy region is reliable? The problem has been discussed in Refs.[13, 25, 26]. In ChQM, the most vector- or axial-vector-dependent processes, like vector and axial-vector meson decays, can be calculated because of the following two reason. 1) Spin-1 meson resonances are introduced in the framework by VMD which is phenomenological result and beyond low energy expansion. 2) ChQM is only a phenomenological model, in which the universal coupling constant  $g^2 = \frac{8}{3} \lambda(\mu_v)$  affects rather many processes. Although the value of  $g$  is fitted phenomenologically at leading order of spin-1 meson resonances coupling to pseudoscalar fields in this paper, in fact, some high order effects of momentum expansion has been included. It will

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<sup>7</sup>Here  $g_\phi$  is fitted only in tree level. In sect. 7, we fit it again in one-loop level and show that the difference is small.

be double counting if we try to discuss high order effects of momentum expansion in terms of  $g$  determined phenomenologically by lagrangian with four derivatives. Hence all high order derivative terms of  $\mathcal{L}^{V,A}$  will be omitted in this paper.

The physical vector and axial-vector fields can be obtained via the following field rescaling in  $\mathcal{L}^{V,A}$  which make kinetic term of spin-1 meson fields into the standard form

$$V_\mu \longrightarrow \frac{1}{g} V_\mu, \quad A_\mu \longrightarrow \frac{1}{g_A} A_\mu, \quad g_A = g \sqrt{1 - \frac{1}{2\pi^2 g^2}} = g\kappa. \quad (3.27)$$

Then due to Eqs.( 3.13), ( 3.15) and ( 3.16) we have

$$\begin{aligned} m_1^2 &= m_V^2 g^2, & \bar{m}_2^2 &= m_A^2 g_A^2, \\ c &= \frac{1 - \sqrt{1 - \frac{4f_0^2}{m_A^2 g_A^2}}}{2}. \end{aligned} \quad (3.28)$$

The above equations require  $m_A^2 g_A^2 \geq 4f_0^2$ .

For fitting the parameters  $g$  and  $c$ , we will calculate on  $\rho$ -mass shell decays  $\rho^0 \rightarrow e^+e^-$  and  $\rho \rightarrow \pi\pi$  in the following. In our calculation, we set  $m_u = m_d = 0$  so that  $f_0 = f_\pi$ .

The  $\rho^0 - \gamma$  vertex reduced by VMD reads from lagrangian ( 3.20) as follows

$$\mathcal{L}_{\rho\gamma} = g_{\rho\gamma}(q^2)\rho_\mu^0 \mathcal{A}^\mu, \quad g_{\rho\gamma}(q^2) = \frac{1}{2}eqq^2, \quad (3.29)$$

where  $\mathcal{A}_\mu$  is photon field. This vertex vanish in  $q^2 \rightarrow 0$  so that it does not violate  $U(1)_{\text{EM}}$  gauge symmetry. Then  $\rho^0 \rightarrow e^+e^-$  decay width can be obtained directly by a photon field exchange

$$\Gamma(\rho^0 \rightarrow e^+e^-) = \frac{\pi}{3}g^2\alpha^2 m_\rho. \quad (3.30)$$

To input experimental data  $6.7 \pm 0.3\text{KeV}$ [27], we can fit  $g = 0.395 \pm 0.008$ .

The  $\rho \rightarrow \pi\pi$  vertex is obtained in the following by means of substituting Eqs.( 3.13) and ( 3.27) into lagrangian ( 3.20)

$$\begin{aligned} \mathcal{L}_{\rho\pi\pi} &= f_{\rho\pi\pi} \epsilon^{ijk} \rho_i^\mu \pi_j \partial_\mu \pi_k, \\ f_{\rho\pi\pi} &= \frac{m_\rho^2}{gf_\pi^2} [2g^2 c (1 - \frac{c}{2}) + \frac{(1-c)^2}{\pi^2}] \end{aligned} \quad (3.31)$$

To take  $c = 0.44$  we obtain

$$\Gamma(\rho \rightarrow \pi\pi) = \frac{f_{\rho\pi\pi}^2}{48\pi} m_\rho (1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}} = (150 \pm 4) \text{MeV}. \quad (3.32)$$

The experimental value is 150 MeV.

Furthermore, the above values of  $g_\phi$ ,  $g$  and  $c$  yield constitute quark mass

$$m = \frac{f_\pi}{g_\phi \sqrt{6(1-c)}} = 320 \text{MeV}, \quad (3.33)$$

and axial-vector meson mass in chiral limit

$$m_A = \frac{f_\pi}{g_A \sqrt{c(1-c)}} = (1154 \pm 6) \text{MeV}. \quad (3.34)$$

The prediction by the second Weinberg sum rule[28] is  $m_A = \sqrt{2}m_\rho = 1090 \text{MeV}$  and experimental data is  $1230 \pm 40 \text{MeV}$ .

There are six free parameters to parameterize the effective lagrangian generated from quark loops. They are  $f_\pi$ (or  $m$ ),  $B_0$ ,  $g_\phi$ ,  $g$ ,  $m_V$  and  $m_A$ (or  $c$ ). These constants determine low energy dynamics of mesons and it is welcome that the number of free parameters is less than ChPT. Approximately, if we redefine logarithmic divergence from quark loop integral in scheme of cutoff regularization, we have

$$\left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \Gamma\left(2 - \frac{D}{2}\right) \simeq \ln\left(1 + \frac{\Lambda^2}{m^2}\right) - \frac{\Lambda^2}{\Lambda^2 + m^2}. \quad (3.35)$$

Then for  $N_c = 3$ , above values of  $g_\phi$  and  $g$  will yield corresponding cut-off  $\Lambda_\phi \sim 1.3 \text{GeV}$  and  $\Lambda_V \sim 2 \text{GeV}$  respectively, where  $\Lambda_\phi$  and  $\Lambda_V$  are cut-offs corresponding  $\mathcal{L}^\phi$  and  $\mathcal{L}^{V.A}$ . It is a very interesting result that  $\Lambda_\phi \sim \Lambda_{CSSB}$  and indicates ChQM is consistent with original discussion on CSSB[29]. In addition, as mentioned in Introduction, it is consistent for  $\Lambda_V > \Lambda_\phi$  and all spin-1 meson masses being below  $\Lambda_V$ .

## 4 Correction of Gluon Coupling, ChQM Versus QCD Sum Rules

### 4.1 Correction of gluon coupling

In principle, correction of gluon interaction can be obtained via integrating over gluon field in ChQM. Unfortunately, we do not know how to perform analytically the remaining integration over the gluonic fields. A available way is to make a weak gluon-field expansion around physical vacuum expectation value and parameterize phenomenologically the contribution.

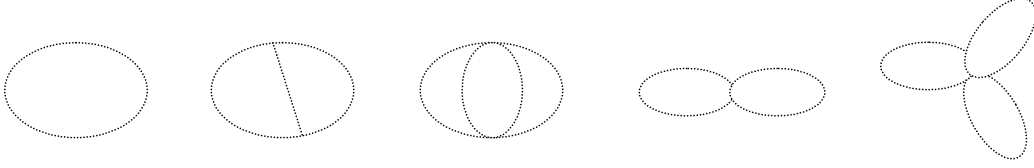
Replacing the operator of Eq.( 3.4) by one including color coupling

$$\begin{aligned} \mathcal{D} = & \gamma^\mu (\partial_\mu - ig_s \frac{\lambda_c^a}{2} G_\mu^a - i \frac{1 - \gamma_5}{2} (L + l)_\mu - i \frac{1 + \gamma_5}{2} (R + r)_\mu) \\ & + mu + (s - ip\gamma_5), \end{aligned} \quad (4.1)$$

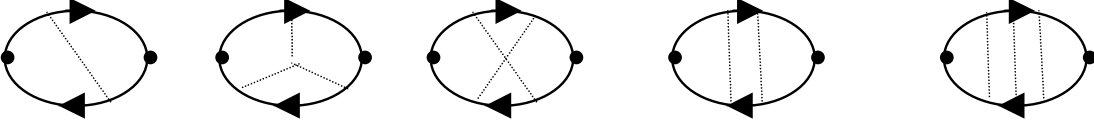
where  $\lambda_c^a (a = 1, 2, \dots, 8)$  denote color SU(3) Gell-Mann matrices. In terms of Eqs.( 3.7)-( 3.9), a effective action including gluon interaction can be obtained via integrating over quark fields. In general, the corresponding effective action with normal parity can be written as

$$I_{Re} = I_M + I_G + I_{MG}. \quad (4.2)$$

Here,  $I_M = \int d^4x \mathcal{L}_{eff}^{(0)}$ , where  $\mathcal{L}_{eff}^{(0)}$  has been obtained in section 3 in which only meson interactions are involved.  $I_G$  is nothing other than the effective action of gluonic fields introduced by the fermion



a) Some diagrams for gluon condensate.



b) Quark-gluon loops corresponding above gluon condensate

Figure 4.1: Some diagrams for gluon condensate in leading order of  $1/N_c$ (fig. a) and its corresponding quark-gluon loops in ChQM(fig. b). Here solid line denotes quark propagator, dot line denotes gluon propagator and "•" denotes other external fields(include meson fields, photon field, etc.).

loop and  $I_{MG}$  includes interaction that gluonic fields couple to mesonic fields. Then we can write the path-integral form for gluonic fields as

$$\begin{aligned}
e^{iI_{eff}} &= \int \mathcal{D}G_\mu^a \cdot \exp \left\{ i \left( -\frac{1}{4} \int d^4x G_{\mu\nu}^a G^{\mu\nu a} + I_M + I_G + I_{MG} \right) \right\} \\
&= \frac{1}{N} e^{iI_M} \int \mathcal{D}G_\mu^a \cdot e^{iI_{MG}} \exp \left\{ i \left( -\frac{1}{4} \int d^4x G_{\mu\nu}^a G^{\mu\nu a} + I_G \right) \right\} \\
&= \frac{1}{N} \exp \left\{ i(I_M + \langle 0|I_{MG}|0 \rangle + \frac{1}{2}(\langle 0|I_{MG}^2|0 \rangle - \langle 0|I_{MG}|0 \rangle^2) + \dots) \right\} \quad (4.3)
\end{aligned}$$

where brackets denote the expectation value of operator. After we trace over color space,  $I_M$  is proportional to  $N_c$  but  $I_{MG}$  is not because  $G_\mu = g_s \frac{\lambda^a}{2} G_\mu^a(x)$  is color- $\lambda^a$ -dependent. Therefore  $I_M$  is  $O(N_c)$  but  $I_{MG}$  is  $O(1/N_c)$  at least due to color coupling constant  $g_s \sim O(1/\sqrt{N_c})$ . However, since expectation value of operator sum over all possible connect loop diagrams of operator, the power counting of  $1/N_c$  become very complicated for  $\langle 0|I_{MG}|0 \rangle$ ,  $\langle 0|I_{MG}^2|0 \rangle$  and  $\langle 0|I_{MG}|0 \rangle^2$  etc.. In general, the numbers of  $O(N_c)$  terms in Eq.( 4.3) are infinite(e.g., see fig. 4.1). Thus it is impossible to sum over contribution of all possible  $O(N_c)$  terms in Eq.( 4.3). A available way is to capture dominant contribution and ignore other minor ingredients.

The success of the QCD sum rules[30] implies that the contribution from quadratic gluon condensate  $\langle 0|(\alpha_s/\pi)G_{\mu\nu}^a G^{a\mu\nu}|0 \rangle$  is dominant at low energy(detail discussion for QCD sum rules is in next subsection). Moreover, it was shown in ref.[15] that contribution of the term with triple



gluon condensate is only around 5% of quadratic one(if we believe the dilute instantons gas estimate of the triple condensate). So that the contribution of gluon coupling will be calculated to  $O(\alpha_s)$  in this paper. Up to this order, the effective action reduced by integrating over gluon fields in  $I_{MG}$  is proportional a constant

$$k = \frac{1}{24m^4} < 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} | 0 > . \quad (4.4)$$

The explicit form of this effective lagrangian is

$$\begin{aligned} \mathcal{L}^{(g)} &= \mathcal{L}_{\text{kin}}^{(g)} + \mathcal{L}_1^{(g)} \\ \mathcal{L}_{\text{kin}}^{(g)} &= \frac{k}{4} m^2 < D_\mu U D^\mu U^\dagger > + k m^3 < \chi U^\dagger + \chi^\dagger U > + \frac{k}{20} < L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} > \\ \mathcal{L}_1^{(g)} &= -\frac{k}{40} < L_{\mu\nu} U^\dagger R^{\mu\nu} U > - \frac{k}{40} < D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger > \\ &\quad + [\frac{km}{8B_0}(1-c-\frac{2m}{B_0}) - \frac{k}{160}(1-c)^2] < \chi U^\dagger \chi U^\dagger + \chi^\dagger U \chi^\dagger U > \\ &\quad - \frac{km}{8B_0} < D_\mu U D^\mu U^\dagger (\chi U^\dagger + \chi^\dagger U) >, \end{aligned} \quad (4.5)$$

where  $\mathcal{L}_{\text{kin}}^{(g)}$  is nothing but to modify free parameters  $f_0$ ,  $B_0$  and  $g_\phi$  or  $g$ . The effective lagrangian  $\mathcal{L}_I^{(g)}$  contribute to low energy coupling constants of ChPT,  $L_3$ ,  $L_5$ ,  $L_8$  and  $L_{10}$ . The results are as follows

$$\begin{aligned} L_3^{(g)} &= -\frac{k}{40}(1-c)^4, \\ L_5^{(g)} &= -\frac{km}{8B_0}(1-c)^2, \\ L_8^{(g)} &= \frac{km}{8B_0}(1-c-\frac{2m}{B_0}) - \frac{k}{160}(1-c)^2, \\ L_{10}^{(g)} &= -\frac{k}{40}(1-c)^2. \end{aligned} \quad (4.6)$$

The above results coincide with one given in Ref.[15] if spin-1 meson resonances disappear. However, in presence of spin-1 meson resonances, these coupling constants are suppressed by factor  $(1-c)$  which come from diagonalization of  $A_\mu - \partial^\mu \Phi$ . Recalling  $c = 0.44$ , we find that here the contribution from gluon coupling is much smaller than one in ref.[15]. In addition, from Eq.( 3.26) we can find that  $L_3^{(0)}$  and  $L_{10}^{(0)}$  are also compensated by exchange effects of spin-1 meson resonances. It make those low energy coupling constants obtained from ChQM agree with ChPT very well(see sect. 7).

The value of gluon condensate has been estimated in QCD sum rules. For determined the value of  $k$  in ChQM, we like to comparing ChQM and QCD sum rules in the following subsection.

## 4.2 ChQM versus QCD sum rules

In this subsection, we will use method of Shifman-Vainshtein-Zakharov(SVZ) sum rules[30] to study relation between ChQM and QCD sum rules in terms of  $\rho$  meson spectral distribution. Review

paper on QCD sum rules of  $\rho$  meson is in [32]. It must be claimed that the comparison is available only for energy scale  $\mu < \Lambda_{\text{CSSB}}$ , since ChQM is legitimate only at low energy.

We start with the time order current-current correlator

$$\Pi_{\mu\nu} = j \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_\mu(x)^{\text{em}} j_\nu(0)^{\text{em}} \} | 0 \rangle, \quad (4.7)$$

where  $q$  is the total momentum of the quark-antiquark pair injected in the vacuum, where  $j_\mu^{\text{em}}$  is electromagnetic current of the  $\rho$  meson,

$$j_\mu^{\text{em}} = \frac{1}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d). \quad (4.8)$$

Due to the current conservation  $\Pi_{\mu\nu}$  is transversal and, hence,

$$\Pi_{\mu\nu} = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2). \quad (4.9)$$

The spectral density of  $\rho$ -meson is defined by

$$\rho(s) = b \text{Im} \Pi(s), \quad s = q^2, \quad (4.10)$$

where normalization constant  $b$  is determined by requirement of Eq.( 4.10) coinciding with the cross-section of  $e^+e^-$  annihilation into hadrons(measured in the units  $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ ). Here we focus our attention on the following integral of spectral density with weight  $e^{-s/M^2}$ ,

$$I(M^2) = \frac{1}{M^2} \int ds e^{-s/M^2} \rho(s). \quad (4.11)$$

The prediction of QCD sum rules for this integral is

$$I(M^2) = I_{\text{npc}}(M^2) + I_{\text{pc}}(M^2), \quad (4.12)$$

where  $I_{\text{npc}}(M^2)$  is non-perturbative contribution and  $I_{\text{pc}}(M^2)$  is correction of perturbative QCD.  $I_{\text{npc}}(M^2)$  was obtained by original work of Shifman *et al.*, with  $m_u = m_d = 0$ [30],

$$\begin{aligned} I_{\text{npc}}(M^2) = & 1 + \frac{\pi^2}{3M^4} \langle 0 | \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{a\mu\nu} | 0 \rangle - \frac{8\pi^2}{M^6} \langle 0 | \alpha_s (\bar{u} \gamma_\alpha \gamma_5 \lambda^a u - \bar{d} \gamma_\alpha \gamma_5 \lambda^a d)^2 | 0 \rangle \\ & - \frac{16\pi^2}{9M^6} \langle 0 | \alpha_s (\bar{u} \gamma_\alpha \lambda^a u + \bar{d} \gamma_\alpha \lambda^a d) \sum_{q=u,d,s} (\bar{q} \gamma_\alpha \lambda^a q) | 0 \rangle, \end{aligned} \quad (4.13)$$

where

$$\begin{aligned} & \langle 0 | \mathcal{O}_G | 0 \rangle \equiv \langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G^{a\mu\nu} | 0 \rangle = x_G 1.2 \times 10^{-2} GeV^4, \\ & \langle 0 | \alpha_s (\bar{u} \gamma_\alpha \gamma_5 \lambda^a u - \bar{d} \gamma_\alpha \gamma_5 \lambda^a d)^2 | 0 \rangle \simeq x_{4q} 6.5 \times 10^{-4} GeV^4, \\ & \langle 0 | \alpha_s (\bar{u} \gamma_\alpha \lambda^a u + \bar{d} \gamma_\alpha \lambda^a d) \sum_{q=u,d,s} (\bar{q} \gamma_\alpha \lambda^a q) | 0 \rangle \simeq -x_{4q} 6.5 \times 10^{-4} GeV^4. \end{aligned} \quad (4.14)$$

In Eq.( 4.14), the parameters  $x_G$  and  $x_{4q}$  are allowed to float in vicinity of unity. This parameterization will be used for matching  $\rho$ -meson sum rule with data.

The perturbative correction up to third order in  $\alpha_s(s)$  is given in Refs.[33, 34]

$$I_{\text{pc}}(M^2) = 1 + \frac{4}{9}a\left(\frac{M^2}{e^\gamma}\right)\{1 + 0.729[a\left(\frac{M^2}{e^\gamma}\right)] - 0.386[a\left(\frac{M^2}{e^\gamma}\right)]^2\}, \quad (4.15)$$

with

$$\begin{aligned} a(\mu^2) &\equiv \left(\frac{11}{12}N_c - \frac{2}{12}N_f\right)\frac{\alpha_s(\mu^2)}{\pi} = \frac{1}{\ln(\mu^2/\Lambda^2)} - 0.79\frac{\ln \ln(\mu^2/\Lambda^2)}{\ln^2(\mu^2/\Lambda^2)} \\ &+ \frac{0.79^2}{\ln^3(\mu^2/\Lambda^2)}[(\ln \ln(\mu^2/\Lambda^2))^2 - \ln \ln(\mu^2/\Lambda^2) + 0.415] + O\left(\frac{1}{\ln^4(\mu^2/\Lambda^2)}\right), \end{aligned} \quad (4.16)$$

where  $\Lambda$  is the scale parameter of QCD introduced in a standard way[35], in this paper we take  $\Lambda = 0.2\text{GeV}$ .

Now let us return to framework of ChQM. Due to VMD, the electromagnetic current of  $\rho$  meson is obtained easily

$$j_\mu^{\text{em}} = \frac{g}{2}(\partial^2 \rho_\mu^0 - \partial_\mu \partial_\nu \rho^{0\nu}). \quad (4.17)$$

Then we obtain polarization operator

$$\Pi(q^2) = \frac{g^2}{4} \frac{q^2}{q^2 - m_\rho^2 + i\sqrt{q^2}\Gamma(q^2)}, \quad (4.18)$$

where  $\Gamma(q^2)$  is just width of  $\rho \rightarrow \pi\pi$  decay when  $q^2 = m_\rho^2$ , so that

$$\begin{aligned} \Gamma(q^2) &= \frac{f_{\rho\pi\pi}^2(q^2)}{48\pi} \sqrt{q^2} \left(1 - \frac{4m_\pi^2}{q^2}\right)^{\frac{3}{2}} \theta(q^2 - 4m_\pi^2), \\ f_{\rho\pi\pi}(q^2) &= \frac{q^2}{gf_\pi^2} \left[2g^2 c \left(1 - \frac{c}{2}\right) + \frac{(1-c)^2}{\pi^2}\right]. \end{aligned} \quad (4.19)$$

It should be pointed out that the momentum-dependence of  $f_{\rho\pi\pi}$  is yielded in this model, since  $\rho - \pi\pi$  vertex come from  $\mathcal{L}_4$  instead of  $\mathcal{L}_2$ . Then the spectral density of  $\rho$ -meson can be written as follows

$$\rho(s) = 2.3\pi g^2 \frac{s\sqrt{s}\Gamma(s)}{(s - m_\rho^2)^2 + s\Gamma^2(s)}, \quad (4.20)$$

where the spectral density has been normalized for coincides with

$$R^{I=1} = \frac{\sigma(e^+e^- \rightarrow \text{hadrons}, I=1)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}.$$

In fig. 4.2 we show the  $I(M^2)$  curve which are obtained from QCD sum rules(without perturbative QCD correction), ChQM and experiment respectively. It is not surprising that the  $I_\chi(M^2)$  obtained from ChQM agree with experiment excellently in  $M^2 < 0.6\text{GeV}^2$  but do not match data when  $M^2 > 1\text{GeV}^2$ . The reason is that ChQM is a low energy model so that other heavier vector-isovector meson resonances, such as  $\rho(1450)$  and  $\rho(1700)$ , are not included in this model. For

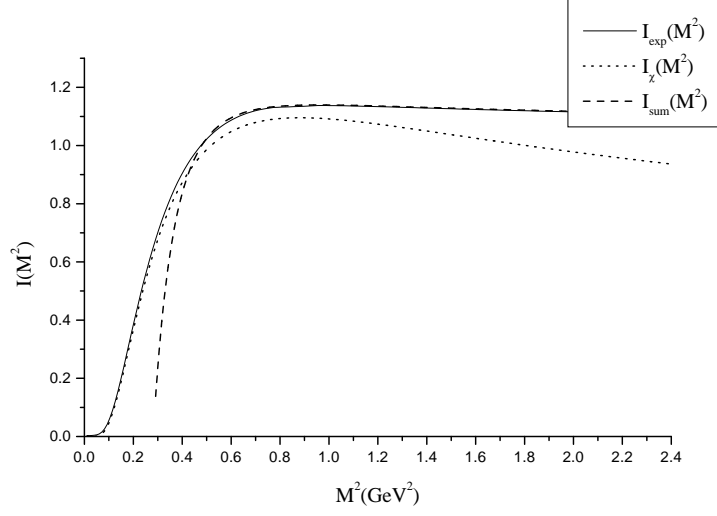


Figure 4.2:  $I(M^2)$  versus  $M^2$  in the  $\rho$ -meson channel: the solid line denotes experiment data, the dot line denotes prediction by ChQM (without perturbative correction) and the dash line denotes prediction by SVZ sum rules with  $x_G = x_{4q} = 1$ (without perturbative correction).

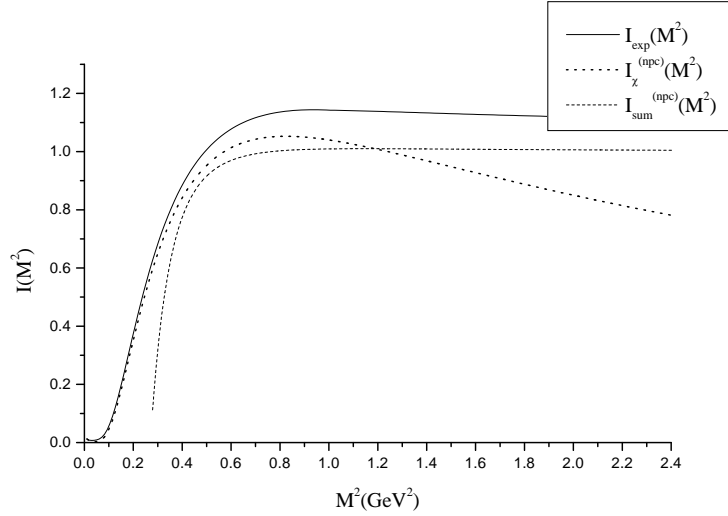


Figure 4.3:  $I(M^2)$  versus  $M^2$  in the  $\rho$ -meson channel: the solid line denotes experiment data, the dot line denotes prediction by ChQM (with perturbative correction) and the dash line denotes prediction by SVZ sum rules with  $x_G = 0.8$ ,  $x_{4q} = 1.3$ (with perturbative correction).

improving this shortage, we like to add perturbative QCD correction into spectral density through the following rough way,

$$\rho(s) = 2.3\pi g^2 \frac{s\sqrt{s}\Gamma(s)}{(s - m_\rho^2)^2 + s\Gamma^2(s)} + 0.4(1 + \frac{\alpha_s(s)}{\pi})\theta(s - s_0), \quad (4.21)$$

where the part of perturbative correction has been normalized the same as non-perturbative part,  $\alpha_s(s)$  can be obtained from Eq.( 4.16) and  $s_0 = 2.5\text{GeV}^2$  is suggested by the experimental data[36]. The result is shown in fig.4.3. We can find now  $I_\chi(M^2)$  agree with data well in  $M^2 < 1.2\text{GeV}^2$ , and indeed perturbative correction in Eq.( 4.21) improve high energy behavior of  $I_\chi(M^2)$  significantly. The  $I_{\text{SVZ}}(M^2)$  obtained from QCD sum rules and including perturbative correction is also shown in fig.4.3, where  $x_G = 0.8$  and  $x_{4q} = 1.3$ . It is shown that ChQM coincides with QCD sum rules at  $\rho$ -meson energy scale. Therefore, the gluon condensate-dependent parameter  $k$  can be determined as follows(with  $x_G = 0.8$  and  $m \simeq 0.32\text{GeV}$ ),

$$k = \frac{\langle 0|\mathcal{O}_G|0 \rangle}{24m^4} = \frac{x_G 0.012\text{GeV}^4}{24m^4} \simeq 0.038. \quad (4.22)$$

Two more remarks are needed here. 1) From fig.4.3 we can find QCD sum rules agree with data excellent when  $M^2 > 0.6\text{GeV}^2$  even without triple or more higher power gluon condensate. It means that quadratic condensate is dominant in gluon vacuum condensate when  $M^2 > 0.6\text{GeV}^2$ . In addition, it can be obtained from Eq.( 4.6) that the quadratic gluon condensate always accompanies the factor  $1/(192m^4)$ . Furthermore, calculation will shown that the triple gluon condensate accompanies the factor  $1/(2880m^6)$  at least. Therefore, in ChQM, the operator product expansion in gluon sector is approximately in powers of  $1/(14m^2)$ . Due to  $14m^2 \simeq 1.4\text{GeV}^2 \gg 0.6\text{GeV}^2$ , the result of QCD sum rules implies that here it is enough to consider quadratic gluon condensate only and to ignore other vacuum condensate with triple or more higher powers of gluon field strength. 2) Although phenomenological comparison between  $I_\chi$  and  $I_{\text{SVZ}}$  is successful, unfortunately, it is open to analytically determine relation between  $I_\chi$  and  $I_{\text{SVZ}}$ . From viewpoint of operator product expansion, it is because there are two extra parameters with dimension 1 in ChQM, constituent quark mass  $m$  and  $\rho$ -meson mass  $m_\rho$ . It make the operator product expansion in framework of ChQM become impracticable. For example, we can construct many operators with dimension 4, such as  $\langle 0|GG|0 \rangle$ ,  $\langle 0|(\bar{\psi}\psi)^2|0 \rangle / m^2$ , .... In general, both of these parameters are functions of vacuum condensate of QCD sum rules. However, the relation between them and vacuum condensate is unknown at all. In fact, it is the most important but the hardest problem in studies on low energy QCD, that we know nothing about how quark and gluon fields become hadrons at low energy.

## 5 General Formula for One Loop of Mesons

For convenience to study one loop contribution of mesons systematically. In this section we like to derive general formula for one loop of mesons with any spin by means of background field method.

Considering an action of mesons  $I[L_\mu, R_\mu, U] = \int d^4x \mathcal{L}(x)$ , we expand meson fields in this action around their classic solutions

$$V_\mu(x) = \bar{V}_\mu(x) + v_\mu(x),$$

$$\begin{aligned} A_\mu(x) &= \bar{A}_\mu(x) + a_\mu(x), \\ U(x) &= \xi e^{i\varphi} \xi, \quad \bar{U}(x) = \xi^2(\Phi), \end{aligned} \quad (5.1)$$

where background fields  $\bar{V}_\mu$ ,  $\bar{A}_\mu$  and  $\bar{U}(x)$  are solutions of classic motion equation of mesons, i.e.,  $\delta I/\delta V_\mu(x) = 0$ ,  $\delta I/\delta A_\mu(x) = 0$  and  $\delta I/\delta U(x) = 0$ , respectively.  $v_\mu(x)$ ,  $a_\mu(x)$  and  $\varphi(x)$  are quantum fluctuation fields around those classic solutions. In following two sections external vector and axial-vector fields are included in background fields  $\bar{V}_\mu$  and  $\bar{A}_\mu$ . Corresponding expansion of fields (5.1), the action is written as

$$I[V_\mu, A_\mu, U] = \bar{I}[\bar{V}_\mu, \bar{A}_\mu, \bar{U}] + \delta I[\bar{V}_\mu, \bar{A}_\mu, \bar{U}; v_\mu, a_\mu, \varphi].$$

Then the quantum correction of previous action,  $\Gamma[\bar{V}_\mu, \bar{A}_\mu, \bar{U}]$ , can be evaluated by means of integrating over the quantum fields

$$e^{i\Gamma[\bar{V}_\mu, \bar{A}_\mu, \bar{U}]} = \int \mathcal{D}v_\mu(x) \mathcal{D}a_\mu(x) \mathcal{D}\varphi(x) e^{i\delta I[\bar{V}_\mu, \bar{A}_\mu, \bar{U}; v_\mu, a_\mu, \varphi]} \quad (5.2)$$

In particular, the one loop contribution is obtained via integrating over the quantum fields in quadratic terms of  $\delta I[\bar{V}_\mu, \bar{A}_\mu, \bar{U}; v_\mu, a_\mu, \varphi]$

$$\begin{aligned} e^{i\Gamma_{one-loop}[\bar{V}_\mu, \bar{A}_\mu, \bar{U}]} &= \int \mathcal{D}v_\mu^a(x) \mathcal{D}a_\mu^a(x) \mathcal{D}\varphi^a(x) \exp\{i \int d^4x (\varphi^a \mathcal{H}_{0\varphi\varphi}^{ab}(x) \varphi^b \\ &\quad + v_\mu^a \mathcal{H}_{0vv}^{\mu\nu,ab}(x) v_\nu^b + a_\mu^a \mathcal{H}_{0aa}^{\mu\nu,ab}(x) a_\nu^b + v_\mu^a \mathcal{H}_{0va}^{\mu\nu,ab}(x) a_\nu^b \\ &\quad + v_\mu^a \mathcal{H}_{0v\varphi}^{\mu,ab}(x) \varphi^b + a_\mu^a \mathcal{H}_{0a\varphi}^{\mu,ab}(x) \varphi^b)\}, \end{aligned} \quad (5.3)$$

where

$$\mathcal{H}_{0st}^{ab}(x) \delta^4(x-y) = \frac{1}{2!} \frac{\delta^2 I}{\delta S^a(x) \delta T^b(y)} \Big|_{V_\mu=\bar{V}_\mu, A_\mu=\bar{A}_\mu, U=\bar{U}}, \quad (5.4)$$

where  $s, t = \varphi, v_\mu, a_\mu$  and  $S, T = \Phi, V_\mu, A_\mu$ . In expansion of the action, the terms to be proportional to quantum field disappear due to classic equation of motion of mesons.

In principle, the integration in Eq.( 5.3) can be performed explicitly in terms of Gauss integral formula, e.g., for two quantum field case

$$\int \mathcal{D}s \mathcal{D}t e^{sAs+tBt+sCt} \propto \frac{1}{\text{Det}(A + CB^{-1}C) \text{Det}B}. \quad (5.5)$$

However, for three quantum field case, the result of functional integral ( 5.3) becomes so complicated that calculations of those determinants are impractical. Instead, like usual treatment in functional integral, we introduce external sources for quantum fields into the action technically for calculating path-integral of Eq.( 5.3).

At first, we consider a lagrangian of bosons, which has general form as follows

$$\begin{aligned} \mathcal{L}(x) &= -\frac{1}{2} \phi_A(x) \nabla_x^{(\phi)AB} \phi_B(x) - \frac{1}{2} \Psi_A(x) \nabla_x^{(\Psi)AB} \Psi_B(x) - \frac{1}{2} \varphi_A(x) \nabla_x^{(\varphi)AB} \varphi_B(x) \\ &\quad + \phi_A(x) \mathcal{H}_\phi^{AB}(x) \phi_B(x) + \Psi_A(x) \mathcal{H}_\Psi^{AB}(x) \Psi_B(x) + \varphi_A(x) \mathcal{H}_\varphi^{AB}(x) \varphi_B(x) \\ &\quad + \phi_A(x) \Omega_{\phi\Psi}^{AB}(x) \Psi_B(x) + \phi_A(x) \Omega_{\phi\varphi}^{AB}(x) \varphi_B(x) + \varphi_A(x) \Omega_{\varphi\Psi}^{AB}(x) \Psi_B(x) \\ &= -\frac{1}{2} \phi_A(x) \nabla_x^{(\phi)AB} \phi_B(x) - \frac{1}{2} \Psi_A(x) \nabla_x^{(\Psi)AB} \Psi_B(x) - \frac{1}{2} \varphi_A(x) \nabla_x^{(\varphi)AB} \varphi_B(x) \\ &\quad + V[\phi(x), \Psi(x), \varphi(x)], \end{aligned} \quad (5.6)$$

where  $\phi(x)$ ,  $\varphi(x)$  and  $\Psi(x)$  are bosons with arbitrary spin and parity, the index  $A$ ,  $B$  may be Lorentz index, gauge group index,...,  $\nabla_{(j)x}(j = \phi, \varphi, \Psi)$  are free field operators of  $j(x)$  fields, e.g.,  $\delta^{ab}(\partial_x^2 + m^2)$  for fields with zero spin and  $\delta^{ab}[\delta^{\mu\nu}(\partial_x^2 + m^2) - \partial_x^\mu \partial_x^\nu]$  for fields with spin-1.  $\mathcal{H}_j(x)$  and  $\Omega_{ij}(x)(i, j = \phi, \varphi, \Psi; i \neq j)$  are function of external fields, and in which differential operators may be included (for our purpose in this paper, it is enough to consider one differential operator included only).  $V[\phi(x), \Psi(x), \varphi(x)]$  denotes interaction between quantum fields and external fields. It is needed to define the adjoint operators of  $\mathcal{H}_j$  and  $\Omega_{ij}$  as follows for arbitrary operators  $F(x)$  and  $G(x)$  in which there are no differential operators

$$\begin{aligned} \int d^4x F(x) [\mathcal{H}_j(x) G(x)] &= \int d^4x [\mathcal{H}_j^*(x) F(x)] G(x) \\ \int d^4x F(x) [\Omega_{ij}(x) G(x)] &= \int d^4x [\Omega_{ij}^*(x) F(x)] G(x) \equiv \int d^4x [\Omega_{ji}(x) F(x)] G(x). \end{aligned} \quad (5.7)$$

where  $\mathcal{H}_j^*$  and  $\Omega_{ij}^* = \Omega_{ji}$  are adjoint operators of  $\mathcal{H}_j$  and  $\Omega_{ij}$  respectively.

The one loop correction of lagrangian (5.6) can be derived by means of path-integral with corresponding external source fields,  $\zeta$ ,  $\eta$  and  $\gamma$

$$\begin{aligned} e^{i\Gamma_{one-loop}} &= \int \mathcal{D}\phi(x) \mathcal{D}\Psi(x) \mathcal{D}\varphi(x) \exp \left\{ i \int d^4x \mathcal{L}(x) \right\} \\ &= \exp \left\{ i \int d^4x V \left[ \frac{1}{i} \frac{\delta}{\delta \zeta(x)}, \frac{1}{i} \frac{\delta}{\delta \eta(x)}, \frac{1}{i} \frac{\delta}{\delta \gamma(x)} \right] \int \mathcal{D}\phi(x) \mathcal{D}\Psi(x) \mathcal{D}\varphi(x) \times \right. \\ &\quad \exp \left\{ i \int d^4x \left[ -\frac{1}{2} \sum_{j=\phi, \Psi, \varphi} j_A(x) \nabla_x^{(j)AB} j_B(x) + \zeta_A(x) \phi^A(x) \right. \right. \\ &\quad \left. \left. + \eta_A(x) \Psi^A(x) + \gamma_A(x) \varphi^A(x) \right] \right\} \Big|_{\zeta=\eta=\gamma=0} \\ &= \frac{1}{N} \exp \left\{ i \int d^4x V \left[ \frac{1}{i} \frac{\delta}{\delta \zeta(x)}, \frac{1}{i} \frac{\delta}{\delta \eta(x)}, \frac{1}{i} \frac{\delta}{\delta \gamma(x)} \right] \right\} e^{iW_\phi[\zeta]} e^{iW_\Psi[\eta]} e^{iW_\varphi[\gamma]} \Big|_{\zeta=\eta=\gamma=0}, \end{aligned} \quad (5.8)$$

where the constant  $N$  do not incorporate dynamics so that we omit it in the following calculation. The generating functional of connect Green function,  $W_\phi[\zeta]$ ,  $W_\Psi[\eta]$  and  $W_\varphi[\gamma]$ , are written as

$$\begin{aligned} W_\phi[\zeta] &= \frac{1}{2} \int d^4x \zeta_A(x) \Delta_\phi^{AB}(x-y) \zeta_B(y) \\ W_\Psi[\eta] &= \frac{1}{2} \int d^4x \eta_A(x) \Delta_\Psi^{AB}(x-y) \eta_B(y) \\ W_\varphi[\gamma] &= \frac{1}{2} \int d^4x \gamma_A(x) \Delta_\varphi^{AB}(x-y) \gamma_B(y) \end{aligned} \quad (5.9)$$

where  $\Delta_j^{AB}(x-y)(j = \phi, \varphi, \Psi)$  are propagators of  $j$  fields (inverse of free field operator  $\nabla_x^{(j)AB}$ ). Substituting Eq.(5.9) into Eq.(5.8) and performing functional differential explicitly, we obtain effective action generated by one loop of fields  $\phi$ ,  $\Psi$  and  $\varphi$  as follows

$$\Gamma_{one-loop} = \sum_{i \neq j} (\Gamma_{one-loop}^{(jj)} + \Gamma_{one-loop}^{(ij)}) + \Gamma_{one-loop}^{(\phi\Psi\varphi)} \quad (j = \varphi, \phi, \Psi) \quad (5.10)$$

where

$$\begin{aligned}
i\Gamma_{one-loop}^{(jj)} &= \int d^4x \mathcal{H}_j^{AB}(x) \Delta_{AB}(x-x) \\
&+ \frac{1}{2} \int d^4x_1 d^4x_2 [\mathcal{H}_j^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [(\mathcal{H}_j^{CD}(x_2) + \mathcal{H}_j^{*DC}(x_2)) \Delta_{DA}^{(j)}(x_2-x_1)] \\
&+ \frac{1}{3} \int d^4x_1 d^4x_2 d^4x_3 [\mathcal{H}_j^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [(\mathcal{H}_j^{CD}(x_2) + \mathcal{H}_j^{*DC}(x_2)) \Delta_{DE}^{(j)}(x_2-x_3)] \\
&\quad \times [(\mathcal{H}_j^{EF}(x_3) + \mathcal{H}_j^{*FE}(x_3)) \Delta_{FA}^{(j)}(x_3-x_1)] \\
&+ \frac{1}{4} \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 [\mathcal{H}_j^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [(\mathcal{H}_j^{CD}(x_2) + \mathcal{H}_j^{*DC}(x_2)) \Delta_{DE}^{(j)}(x_2-x_3)] \\
&\quad \times [(\mathcal{H}_j^{EF}(x_3) + \mathcal{H}_j^{*FE}(x_3)) \Delta_{FG}^{(j)}(x_3-x_4)] [(\mathcal{H}_j^{GH}(x_4) + \mathcal{H}_j^{*HG}(x_4)) \Delta_{HA}^{(j)}(x_4-x_1)] \\
&+ \dots, \quad (j = \phi, \varphi, \Psi) \tag{5.11}
\end{aligned}$$

$$\begin{aligned}
i\Gamma_{one-loop}^{(ij)} &= \frac{1}{2} \int d^4x_1 d^4x_2 [\Omega_{ij}^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [\Omega_{ji}^{DC}(x_2) \Delta_{DA}^{(i)}(x_2-x_1)] \\
&+ \frac{1}{2} \int d^4x_1 d^4x_2 d^4x_3 \sum_{k=i,j} [\Omega_{ij}^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [\Omega_{ji}^{DC}(x_2) \Delta_{DE}^{(i)}(x_2-x_3)] \\
&\quad \times [(\mathcal{H}_k^{EF}(x_3) + \mathcal{H}_k^{*FE}(x_3)) \Delta_{FA}^{(k)}(x_3-x_1)] \\
&+ \frac{1}{4} \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 [\Omega_{ij}^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [\Omega_{ji}^{DC}(x_2) \Delta_{DE}^{(i)}(x_2-x_3)] \\
&\quad \times [\Omega_{ij}^{EF}(x_3) \Delta_{FG}^{(j)}(x_3-x_4)] [\Omega_{ji}^{HG}(x_4) \Delta_{HA}^{(i)}(x_4-x_1)] \\
&+ \frac{1}{2} \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 [\Omega_{ij}^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [(\mathcal{H}_j^{CD}(x_2) + \mathcal{H}_j^{*DC}(x_2)) \Delta_{DE}^{(j)}(x_2-x_3)] \\
&\quad \times [\Omega_{ji}^{FE}(x_3) \Delta_{FG}^{(i)}(x_3-x_4)] [(\mathcal{H}_i^{GH}(x_4) + \mathcal{H}_i^{*HG}(x_4)) \Delta_{HA}^{(i)}(x_4-x_1)] \\
&+ \frac{1}{2} \int d^4x_1 d^4x_2 d^4x_3 d^4x_4 \sum_{k=i,j} [\Omega_{ij}^{AB}(x_1) \Delta_{BC}^{(j)}(x_1-x_2)] [\Omega_{ji}^{DC}(x_2) \Delta_{DE}^{(i)}(x_2-x_3)] \\
&\quad \times [(\mathcal{H}_k^{EF}(x_3) + \mathcal{H}_k^{*FE}(x_3)) \Delta_{FG}^{(k)}(x_3-x_4)] [(\mathcal{H}_k^{GH}(x_4) + \mathcal{H}_k^{*HG}(x_4)) \Delta_{HA}^{(k)}(x_4-x_1)] \\
&+ \dots \quad (i \neq j) \tag{5.12}
\end{aligned}$$

$$\begin{aligned}
i\Gamma_{one-loop}^{(\phi\Psi\varphi)} &= \int d^4x_1 d^4x_2 d^4x_3 [\Omega_{\phi\varphi}^{AB}(x_1) \Delta_{(\varphi)BC}(x_1-x_2)] [\Omega_{\varphi\Psi}^{CD}(x_2) \Delta_{(\Psi)DE}(x_2-x_3)] \\
&\quad \times [\Omega_{\Psi\phi}^{EF}(x_3) \Delta_{(\phi)FA}(x_3-x_1)] + \dots \tag{5.13}
\end{aligned}$$

Here  $\Gamma_{one-loop}^{(jj)}$  denotes the effective action generated by one loop in which all internal lines are propagators of  $j$  field (see fig. 6.1). Hereafter we call this kind of loops as “pure loop” of  $j$  field.  $\Gamma_{one-loop}^{(ij)}$  denotes the effective action generated by one loop with internal lines of two different fields  $i, j$  (see fig. 6.1). To call it as “mixing loop” of  $i, j$  fields hereafter. Similarly,  $\Gamma_{one-loop}^{(\phi\Psi\varphi)}$  denotes effective action generated by mixing loops with internal lines of three different fields.



An useful formula is as follows. If  $F_1(x)$ ,  $F_2(x)$ , ...,  $F_n(x)$  and  $G_1(x)$ ,  $G_2(x)$ , ...,  $G_n(x)$  are operators in D dimension spacetime and are independent on differential operator. Their Fourier transformation are

$$F_i(x) = \int \frac{d^D p}{(2\pi)^D} f_i(p) e^{-ip \cdot x} \quad G_i(x) = \int \frac{d^D p}{(2\pi)^D} g_i(p) e^{-ip \cdot x}. \quad (5.14)$$

In addition,  $\Delta(x - y)$  is propagator of a field and  $\Delta(p)$  is its Fourier transformation,

$$\Delta(x - y) = \int \frac{d^D p}{(2\pi)^D} \Delta(p) e^{-ip \cdot (x - y)} \quad (5.15)$$

Then we have the following formula which is available in low energy region.

$$\begin{aligned} & \int d^D x_1 d^D x_2 \dots d^D x_n [(F_1^{A_1}(x_1) \frac{\partial}{\partial x_1^{A_1}} + G_1(x_1)) \Delta(x_1 - x_2)] \times \\ & [(F_2^{A_2}(x_2) \frac{\partial}{\partial x_2^{A_2}} + G_1(x_2)) \Delta(x_2 - x_3)] \dots [(F_n^{A_n}(x_n) \frac{\partial}{\partial x_n^{A_n}} + G_1(x_n)) \Delta(x_n - x_1)] \\ = & \int d^D x \int \frac{d^D q}{(2\pi)^D} [F_1^{A_1}(x) (\frac{\partial}{\partial x^{A_1}} + iq_{A_1}) + G_1(x)] \Delta(q - i\partial) \\ & \times [F_2^{A_2}(x) (\frac{\partial}{\partial x^{A_2}} + iq_{A_2}) + G_2(x)] \Delta(q - i\partial) \dots [F_{n-1}^{A_{n-1}}(x) (\frac{\partial}{\partial x^{A_{n-1}}} + iq_{A_{n-1}}) + G_{n-1}(x)] \\ & \Delta(q - i\partial) [F_n^{A_n}(x) (\frac{\partial}{\partial x^{A_n}} + iq_{A_n}) + G_n(x)] \Delta(q - i\partial), \end{aligned} \quad (5.16)$$

where the formal operator symbol  $\Delta(q - i\partial)$  has been introduced, which can be expanded in power of external momentum in low energy region. For instance, for zero spin fields it is

$$\Delta(q - i\partial) \propto \frac{1}{q^2 - m^2 + i\epsilon} (1 + \frac{\partial^2 + 2iq \cdot \partial}{q^2 - m^2 + i\epsilon} - \frac{4(q \cdot \partial)^2}{(q^2 - m^2)^2 + i\epsilon} + \dots) \quad (5.17)$$

and for spin-1 fields it is

$$\begin{aligned} \Delta_{\mu\nu}(q - i\partial) &= \Delta_{\mu\nu}(q) + \Delta_{\mu\rho}(q) \mathcal{P}^{\rho\sigma} \Delta_{\sigma\nu}(q) + \Delta_{\mu\rho}(q) \mathcal{P}^{\rho\sigma} \Delta_{\sigma\lambda}(q) \mathcal{P}^{\lambda\tau} \Delta_{\tau\nu}(q) + \dots, \\ \Delta_{\mu\nu}(q) &\propto \frac{1}{q^2 - m^2 + i\epsilon} (\delta_{\mu\nu} + \frac{q_\mu q_\nu}{q^2 - m^2}) \\ \mathcal{P}_{\mu\nu} &= (\partial^2 + 2iq \cdot \partial) \delta_{\mu\nu} + \partial_\mu \partial_\nu \end{aligned} \quad (5.18)$$

The formula can be checked directly by inserting Eqs.( 5.14) and ( 5.15) into left side of Eq.( 5.16). Employing the formula( 5.16) in Eq.( 5.11)-( 5.13), the effective action  $\Gamma_{one-loop}$  can be expressed by explicit integral of loop-momentum(to see appendix). It should be pointed out that low energy theorem allows to expand integrand in Feynman-integral of meson-loops in powers of external momentum. For example, setting  $k = q + p$  (where  $p$  is external momentum,  $k$  and  $q$  are momentums carried by internal lines of loop), then we have

$$\frac{1}{(q^2 + m^2)((k^2 + m^2))} = \frac{1}{(q^2 + m^2)^2} + \frac{2q \cdot p + p^2}{(q^2 + m^2)^3} + \frac{4(q \cdot p)^2}{(q^2 + m^2)^4} + \dots \quad (5.19)$$

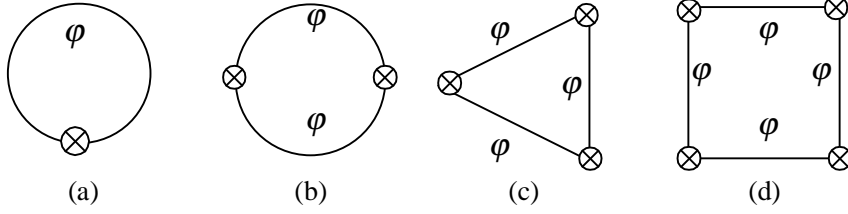


Figure 6.1: Pure one-loop graphs of  $0^-$  mesons. Here “ $\otimes$ ” denotes classic external fields and  $\varphi$  denotes internal lines of  $0^-$  mesons.

## 6 One Loop of Mesons in ChQM

It has been well known that, at very low energy, the effective lagrangian  $\mathcal{L}_2^\phi$  is not only treated at tree level but its one-loop graphs contribute to  $\mathcal{L}_4^\phi$ . Simultaneously,  $\mathcal{L}_4^\phi$  is treated at tree level only [1, 9, 10] because its one-loop contribution belongs to higher order terms only. These conclusion and their proof are still suited for here since low energy expansion on  $\mathcal{L}^\phi$  is well-defined in ChQM. It will be checked again in section 6.1 by another method. Obviously, besides pseudoscalar, in this framework internal line of one-loop graphs can be spin-1 meson fields too. It has been pointed out that, effective lagrangian  $\mathcal{L}^{V,A}$  with six or higher derivative terms is not well-defined because the parameter  $g$  is determined phenomenologically by leading order of spin-1 meson coupling to pseudoscalar fields. Therefore in this paper we only consider spin-1 meson-dependent one-loop contribution generated by  $\mathcal{L}^{V,A}$  with four derivatives, i.e., leading coupling of spin-1 mesons and pseudoscalar mesons.

There are several remarks relating to our following calculations: 1) Since the contribution from gluon coupling is much smaller than one from quark loops, we omit the one-loop effects generated by lagrangian (4.5). 2) Pseudoscalar mesons are treated as massless particles for keeping chiral symmetry of results. 3) In following discussion we appoint that  $p$  is external momentum and  $q$  is momentum carried by internal lines (hereafter we call it as loop-momentum). Due to Eq. (5.19), it is convenience in following analysis that we use a momentum to label all momentums carried by every internal lines, because the difference is only high order of external momentum, and which can be taken into account in calculation on high order diagram. 4) In our calculation the dimensional regularization is used for keeping chiral symmetry. However, sometimes for convenience we will approximately use cut-off regularization to discuss problems.

### 6.1 Pure one-loops of pseudoscalar mesons

To  $O(p^4)$  there are four kinds of potential one-loop graphs which involve pseudoscalar meson internal lines only (fig. 6.1)

1. Recalling that we treat pseudoscalar as zero mass particles for keeping chiral symmetry, the potential "tadpole" contribution (fig. 6.1-a) can be omitted because of in dimensional

regularization

$$\int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 + i\epsilon} \equiv 0. \quad (6.1)$$

2. If there are some vertices generated by  $\mathcal{L}_4$  in figure 6.1-b, 6.1-c and 6.1-d, the internal lines in these figures must carry momentum since external momentum cannot be higher than  $p^4$ . It make integral form of one loop be same as Eq.( 6.1) or yield more high order divergences(higher than quadratic). These contribution can be omitted in dimensional regularization too. Therefore merely one-loop graphs generated by pseudoscalar mesons in  $\mathcal{L}_2$  are needed to calculate in this paper. This conclusion is same as one obtained by Weinberg power counting rules[1, 9].

Inserting Eq.( 5.1) in  $\mathcal{L}_2$ ( 3.15) and retaining terms to quadratic form of quantum fields we obtain

$$\mathcal{L}_2 = \bar{\mathcal{L}}_2 + \frac{f_0^2}{16} < d_\mu \varphi d^\mu \varphi - [\Delta_\mu, \varphi][\Delta^\mu, \varphi] - \frac{1}{4} \{ \varphi, \varphi \} (\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) > \quad (6.2)$$

where the antihermitian matrix  $\Delta_\mu$  and  $\Gamma_\mu$  and covariant derivative have been defined in sect. 3.1.

Employing the completeness relation of generators  $\lambda^a (a = 1, 2, \dots, N^2 - 1)$  of  $SU(N)$ [9]

$$\begin{aligned} < \lambda^a A \lambda^a B > = -\frac{2}{N} < AB > + 2 < A > < B > \\ < \lambda^a A > < \lambda^a B > = 2 < AB > - \frac{2}{N} < A > < B > \end{aligned} \quad (6.3)$$

we can write the vertex included quadratic form of  $\varphi$  in more explicit form in terms of the components  $\varphi^a$

$$\begin{aligned} \mathcal{L}_{\varphi\varphi}(x) &= \varphi^a \mathcal{H}_{0\varphi\varphi}^{ab}(x) \varphi^b = -\frac{1}{2} \varphi^a(x) \nabla_x^{(\varphi)ab} \varphi^b(x) + \varphi^a(x) \mathcal{H}_\varphi^{ab}(x) \varphi^b(x) \\ \nabla^{ab} &= \frac{f_0^2}{4} \delta^{ab} \partial^2 \\ \mathcal{H}_\varphi^{ab} &= -\frac{f_0^2}{8} (\{ \partial^\mu, \Gamma_\mu^{ab} \} + \Gamma_\mu^{ac} \Gamma_\mu^{cb}) - \frac{f_0^2}{16} < [\Delta_\mu, \lambda^a][\Delta^\mu, \lambda^b] > \\ &\quad - \frac{f_0^2}{64} < \{ \lambda^a, \lambda^b \} (\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) > \end{aligned} \quad (6.4)$$

where

$$\Gamma_\mu^{ab} = -\frac{1}{2} < [\lambda^a, \lambda^b] \Gamma_\mu > . \quad (6.5)$$

From Eq.( 6.4) we can find that the adjoint operator of  $\mathcal{H}_\varphi^{ab}$  defined in sect. 5 is

$$\mathcal{H}_\varphi^{*ab} = \mathcal{H}_\varphi^{ba} \quad (6.6)$$

The free field operator  $\nabla_x^{(\varphi)ab}$  in Eq.( 6.4) yield propagator of  $\varphi$  field

$$\Delta^{(\varphi)ab}(x-y) = \int \frac{d^4 q}{(2\pi)^4} \delta^{ab} \Delta^{(\varphi)}(q) e^{-iq \cdot (x-y)}, \quad \Delta^{(\varphi)}(q) = -\frac{4}{f_0^2} \frac{1}{q^2 + i\epsilon} \quad (6.7)$$

In fact, due to massless pseudoscalar fields, there are infrared divergences when we substitute the propagator of  $\varphi$ ( 6.7) into Eq.( 8.2) and perform the integral of loop-momentum explicitly. Because the effective lagrangian generated by one-loops of pseudoscalar mesons is  $O(p^4)$ , we can introduce a external momentum scale factor  $\mu_p$  to regularize this infrared divergence, i.e.,

$$\frac{1}{q^2 + i\epsilon} \longrightarrow \frac{1}{q^2 + \mu_p^2 + i\epsilon} = \frac{1}{q^2 + i\epsilon} \left(1 - \frac{\mu_p^2}{q^2 + i\epsilon} + \dots\right) \quad \mu_p^2 \simeq m_\phi^2 \quad (6.8)$$

From above equation we can easily find that the contribution of  $\mu_p^2$  is  $O(p^6)$ , so that above method to regularize infrared divergences is available.

Now substituting the vertex( 6.4), and propagator ( 6.8) into general formula( 8.2) and performing the integral of loop momentum, we obtain

$$\mathcal{L}_{one-loop}^{(\varphi\varphi)} = \frac{1}{4(4\pi)^{D/2}} \left(\frac{\mu^2}{\mu_p^2}\right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \left(\frac{1}{6} \Gamma_{\mu\nu}^{ab} \Gamma_{\mu\nu}^{ba} + \sigma^{ab} \sigma^{ba}\right), \quad (6.9)$$

where

$$\begin{aligned} \Gamma_{\mu\nu}^{ab} &= -\frac{1}{2} \langle \Gamma_{\mu\nu}[\lambda^a, \lambda^b] \rangle \\ \Gamma_{\mu\nu} &= \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu] = [d_\mu, d_\nu] \\ &= -[\Delta_\mu, \Delta_\nu] - \frac{i}{2} (\xi F_{\mu\nu}^L \xi^\dagger + \xi^\dagger F_{\mu\nu}^R \xi) \\ \sigma^{ab} &= \frac{1}{2} \langle [\Delta^\mu, \lambda^a][\Delta_\mu, \lambda^b] \rangle + \frac{1}{8} \langle \{\lambda^a, \lambda^b\}(\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) \rangle \end{aligned} \quad (6.10)$$

Then we can insert Eqs.( 6.10) in Eq.( 6.9) and simplify the result by employing completeness relation of generators of  $SU(3)$ (Eq.( 6.3),  $N = 3$ ) and the following identity[9]

$$\langle ABAB \rangle = -2 \langle A^2 B^2 \rangle + \frac{1}{2} \langle A^2 \rangle \langle B^2 \rangle + \langle AB \rangle^2. \quad (6.11)$$

In explicit form, the effective lagrangian generated by one-loop graphs in fig. 6.1 reads

$$\begin{aligned} \mathcal{L}_{one-loop}^{(\varphi\varphi)} &= \frac{1}{4(4\pi)^{D/2}} \left(\frac{\mu^2}{\mu_p^2}\right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \left\{ -\frac{1}{4} \langle F_{\mu\nu}^L F^{L\mu\nu} + F_{\mu\nu}^R F^{R\mu\nu} + 2F_{\mu\nu}^L \bar{U}^\dagger F^{R\mu\nu} \bar{U} \rangle \right. \\ &\quad - \frac{i}{2} \langle F^{L\mu\nu} \nabla_\mu \bar{U}^\dagger \nabla_\nu \bar{U} + F^{R\mu\nu} \nabla_\mu \bar{U} \nabla_\nu \bar{U}^\dagger \rangle + \frac{3}{16} \langle \nabla_\mu \bar{U}^\dagger \nabla^\mu \bar{U} \rangle^2 \\ &\quad + \frac{3}{8} \langle \nabla_\mu \bar{U}^\dagger \nabla_\nu \bar{U} \rangle \langle \nabla^\mu \bar{U}^\dagger \nabla^\nu \bar{U} \rangle \\ &\quad + \frac{1}{4} \langle \nabla_\mu \bar{U}^\dagger \nabla^\mu \bar{U} \rangle \langle \chi \bar{U}^\dagger + \chi^\dagger \bar{U} \rangle + \frac{3}{4} \langle \nabla_\mu \bar{U}^\dagger \nabla^\mu \bar{U} (\chi \bar{U}^\dagger + \chi^\dagger \bar{U}) \rangle \\ &\quad \left. + \frac{11}{72} \langle \chi \bar{U}^\dagger + \chi^\dagger \bar{U} \rangle^2 + \frac{5}{24} \langle \chi \bar{U}^\dagger \chi \bar{U}^\dagger + \chi^\dagger \bar{U} \chi^\dagger \bar{U} \rangle \right\} \end{aligned} \quad (6.12)$$

The effective lagrangian ( 6.12) is just result of ChPT[9, 24]. Of course, in framework of truncated field theory, it is different from ChPT that the divergences in Eq.( 6.12) have to be parameterized, i.e., we have to define

$$g_1 = \frac{1}{(4\pi)^{D/2}} \left(\frac{\mu^2}{\mu_p^2}\right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \quad (6.13)$$

to absorb the divergence in Eq.( 6.12). It also means that a cut-off for pseudoscalar meson loop integral is introduced here. Phenomenologically, contribution from meson loops should be small than one from quark loops. We can find all coefficients in lagrangian ( 6.12) are not small sufficient to provide suppression. Thus the parameter  $g_1$  must be smaller than  $g_\phi^2$  which absorb the divergences from quark loops. Usually, it is argued simply that  $g_1$  is suppressed by  $1/N_c$  due to  $g_1/g_\phi^2 \sim O(1/N_c)$ . In practice, the parameter  $g_1$  should be very small. For instance, comparing with experimental data of  $L_1$  we have  $g_1 \ll 64L_1/3 \simeq 0.015$ . However, for  $N_c = 3$  in real world,  $1/N_c$  expansion can not yield a suppression for  $g_1$  as large as our expectation. In fact, the scale also play important role in calculation of meson loops. It means that the truncated point in meson loops should be smaller than one in quark loops. Approximately, if we suppose  $\mu_p \rightarrow m_\pi$  in very low energy limit and define  $g_1$  in scheme of cut-off regularization

$$g_1 = \frac{1}{16\pi^2} \left[ \ln \left( 1 + \frac{\Lambda^2}{\mu_p^2} \right) - \frac{\Lambda^2}{\Lambda^2 + \mu_p^2} \right]. \quad (6.14)$$

Then we have  $\Lambda \ll 700\text{MeV}$ . This cut-off is much smaller than one from quark loops. It indicates square of momentum transfer is very small in pure pseudoscalar meson one-loop. The similar result will be obtained in other meson one-loop calculation.

Finally, we point out that flavor number do not play crucial role in effective lagrangian reduced by meson loops. We can perform calculation in  $N$  flavors and take  $N = 3$  finally.

## 6.2 Pure one-loops of spin-1 mesons

Inserting Eq.( 5.1) into  $\mathcal{L}^{V,A}$  and retaining terms to quadratic form of quantum fields  $v_\mu$  or  $a_\mu$  we obtain

$$\begin{aligned} \mathcal{L}_{(vv)} = & -\frac{1}{4} \langle (\bar{d}_\mu v_\nu - \bar{d}_\nu v_\mu)^2 \rangle + \frac{1}{8} i \langle [v_\mu, v_\nu] (\xi \bar{L}^{\mu\nu} \xi^\dagger + \xi^\dagger \bar{R}^{\mu\nu} \xi) \rangle \\ & - \frac{\kappa^2}{4} \langle ([\bar{\Delta}_\mu, v_\nu] - [\bar{\Delta}_\nu, v_\mu]) [\bar{\Delta}^\mu, v^\nu] \rangle + \frac{4\gamma}{3} \langle [v_\mu, v_\nu] [\bar{\Delta}^\mu, \bar{\Delta}^\nu] \rangle \\ & + \frac{1}{4} m_V^2 \langle v_\mu v^\mu \rangle \end{aligned} \quad (6.15)$$

and

$$\begin{aligned} \mathcal{L}_{(aa)} = & -\frac{1}{4} \langle (\bar{d}_\mu a_\nu - \bar{d}_\nu a_\mu)^2 \rangle + \left( \frac{1}{8\kappa^2} - \frac{2\gamma}{3g_A^2} \right) i \langle [a_\mu, a_\nu] (\xi \bar{L}^{\mu\nu} \xi^\dagger + \xi^\dagger \bar{R}^{\mu\nu} \xi) \rangle \\ & + \left( \frac{8\gamma}{3g_A^2} - \frac{1}{4\kappa^2} \right) \langle ([\bar{\Delta}_\mu, a_\nu] - [\bar{\Delta}_\nu, a_\mu]) [\bar{\Delta}^\mu, a^\nu] \rangle \\ & + \frac{2\theta}{g_A^2} \langle \{a_\mu, a_\mu\} (\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) \rangle \\ & - \frac{4\gamma}{3g_A^2} \langle \{a_\mu, a_\mu\} \{ \bar{\Delta}_\mu, \bar{\Delta}_\nu \} + 2\bar{\Delta}_\mu a_\nu \bar{\Delta}^\mu a^\nu \rangle + \frac{1}{4} m_A^2 \langle a_\mu a^\mu \rangle, \end{aligned} \quad (6.16)$$

where  $g_A$  and  $\kappa$  have been define in Eq.( 3.27) and

$$\theta = \theta_1 = \frac{3g^2 m}{16B_0} - \frac{\gamma m}{2B_0}$$

$$\begin{aligned}
\bar{d}_\mu t_\nu &= d_\mu t_\nu - i[\bar{V}, t_\mu], \quad (t = v, a) \\
\bar{\Delta}_\mu &= (1 - c)\Delta_\mu - i\bar{A}_\mu = \frac{1}{2}\xi^\dagger D_\mu U \xi^\dagger = -\frac{1}{2}\xi D_\mu U \xi, \\
\bar{L}_{\mu\nu} &= (1 - \frac{c}{2})F_{\mu\nu}^L + \frac{c}{2}F_{\mu\nu}^R + \xi^\dagger(\bar{V}_{\mu\nu} - \bar{A}_{\mu\nu})\xi - 2ic(1 - \frac{c}{2})\xi^\dagger[\bar{\Delta}_\mu, \bar{\Delta}_\nu]\xi \\
&\quad - (1 - c)\xi^\dagger([\bar{\Delta}_\mu, \bar{V}_\nu - \bar{A}_\nu] - [\bar{\Delta}_\nu, \bar{V}_\mu - \bar{A}_\mu])\xi \\
\bar{R}_{\mu\nu} &= (1 - \frac{c}{2})F_{\mu\nu}^R + \frac{c}{2}F_{\mu\nu}^L + \xi(\bar{V}_{\mu\nu} + \bar{A}_{\mu\nu})\xi^\dagger - 2ic(1 - \frac{c}{2})\xi[\bar{\Delta}_\mu, \bar{\Delta}_\nu]\xi^\dagger \\
&\quad + (1 - c)\xi([\bar{\Delta}_\mu, \bar{V}_\nu + \bar{A}_\nu] - [\bar{\Delta}_\nu, \bar{V}_\mu + \bar{A}_\mu])\xi^\dagger
\end{aligned} \tag{6.17}$$

with

$$\begin{aligned}
\bar{V}_{\mu\nu} &= d_\mu \bar{V}_\nu - d_\nu \bar{V}_\mu - i[\bar{V}_\mu, \bar{V}_\nu] - i[\bar{A}_\mu, \bar{A}_\nu] \\
\bar{A}_{\mu\nu} &= d_\mu \bar{A}_\nu - d_\nu \bar{A}_\mu - i[\bar{A}_\mu, \bar{V}_\nu] - i[\bar{V}_\mu, \bar{A}_\nu].
\end{aligned} \tag{6.18}$$

The kinetic terms of quantum fields  $v_\mu$  and  $a_\mu$  in Eqs.( 6.15) and ( 6.16) yield their propagators in momentum space as follows

$$\Delta_{\mu\nu}^{(t)}(q) = \frac{1}{q^2 - m_T^2 + i\epsilon}(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{m_T^2}), \tag{6.19}$$

where  $t = v, a$  and  $T = V, A$ . It seem that the propagator of spin-1 mesons yield bad ultraviolet behavior in loop-momentum integral. However, it is only ostensible conclusion in the present framework. For illustrating it, we like to study kinetic term of vector mesons generated by meson one-loop. The completeness relation of generators  $\lambda^a (a = 1, 2, \dots, N^2 - 1)$  of  $SU(N)$  ( 6.3), vertices ( 6.15), ( 6.16) and chiral symmetry require kinetic term of vector mesons generated by vector meson one-loop has form as follows

$$f(\frac{\Lambda^2}{m_V^2})[c_1 N_f < \bar{V}_{\mu\nu} \bar{V}^{\mu\nu} > - c_1 < \bar{V}_{\mu\nu} > < \bar{V}^{\mu\nu} >] \tag{6.20}$$

where for convenience to discuss, we introduce ultraviolet cut-off to regularize divergence, and all possible high order powers of  $\Lambda$  are also included in  $f(\frac{\Lambda^2}{m_V^2})$ . The kinetic term of vector mesons generated by axial-vector meson one-loop has similar form. OZI rule indicates the coefficient of  $< \bar{V}_{\mu\nu} > < \bar{V}^{\mu\nu} >$  should be very small comparing with  $g^2 \simeq 0.16$ . However, since the coefficient of  $\xi \bar{L}^{\mu\nu} \xi^\dagger + \xi^\dagger \bar{R}^{\mu\nu} \xi$  in vertices ( 6.15) are not small, the constants  $c_1$  are not small sufficient too. Therefore, factor  $f(\frac{\Lambda^2}{m_V^2})$  should be very small here(similarly,  $f(\frac{\Lambda^2}{m_A^2})$  is very small simultaneously).

In this present model, it happen only for very low energy cut-off(i.e.,  $f(\frac{\Lambda^2}{m_T^2})$  vanish for  $\Lambda \rightarrow 0$ , where  $T = V, A$ ). It means that square of the momentum transfer of meson loop is small even for higher energy scale(e.g.,  $\mu \sim m_\rho$ ). Sequentially,  $q_\mu q_\nu / m_T^2$  in vector and axial-vector propagators is suppressed due to small momentum transfer. Thus this terms can be omitted here and high order divergences will disappear.

Instead of above intuitive analysis, we can discuss this problem through another way. To consider operator equation of quantum field  $t_\mu(t = v, a)$

$$\frac{\delta \mathcal{L}}{\delta t_\mu} = 0. \quad (6.21)$$

We writing this equation in explicit form

$$d_\nu t^{\mu\nu} + m_T^2 t^\mu = j_T^\mu(v, a, \varphi). \quad (6.22)$$

Acting covariant differentiators  $d_\mu$  to two side of above equation, we obtain

$$d_\mu t^\mu = -\frac{1}{m_T^2} d_\mu \bar{j}_T^\mu. \quad (6.23)$$

On the other hand, acting  $d_\nu$  to two side of Eq.( 6.22), we obtain

$$\partial^2 d_\mu \bar{t}_\nu + m_T^2 d_\mu t_\nu = d_\mu \tilde{j}_\nu^T. \quad (6.24)$$

Here number of derivatives in  $j_T^\mu$ ,  $\bar{j}_T^\mu$  and  $\tilde{j}_T^\mu$  are the same. Solution of equation( 6.24) is

$$d_\mu \bar{t}_\nu(x) = \int d^4 y \Delta^{(T)}(x - y) \tilde{j}_{\mu\nu}^T(y) \quad (6.25)$$

where

$$\begin{aligned} \tilde{j}_{\mu\nu}^T(y) &= d_\mu \tilde{j}_\nu^T(y) \\ \Delta^{(T)}(x - y) &= \int \frac{d^4 p}{(2\pi)^4} \Delta^{(T)}(p) e^{ip \cdot (x-y)}, \quad \Delta^{(T)}(p) = \frac{-1}{p^2 - m_T^2}. \end{aligned} \quad (6.26)$$

Comparing Eq.( 6.23) with Eq.( 6.25) we can find that the Lorentz covariant term  $d_\mu t^\mu$  is high order of momentum expansion comparing with  $d_\mu t_\nu$ . Therefore the terms  $\langle (d_\mu t^\mu)^2 \rangle$  can be omitted in our calculation.

Applying above argument in vertices( 6.15) and ( 6.16) we have

$$\begin{aligned} \mathcal{L}_{(tt)} &= \frac{1}{2} t_\mu^a d_\nu^{ac} d^{\nu, cb} t_\mu^b + t_\mu^a \mathcal{S}_t^{\mu\nu, ab} t_\nu^b + t_\mu^a \mathcal{A}_t^{\mu\nu, ab} t_\nu^b + \frac{m_t^2}{2} t_\mu^a t^{\mu a} \\ &= -\frac{1}{2} t_\mu^a \nabla^{(t)\mu\nu} t_\nu^a + t_\mu^a \mathcal{H}_t^{\mu\nu, ab} t_\nu^b, \quad (t = v, a), \end{aligned} \quad (6.27)$$

where  $\mathcal{S}_t^{\mu\nu, ab}$  is symmetrical and  $\mathcal{A}_t^{\mu\nu, ab}$  is antisymmetrical under interchange of Lorentz index  $\mu$  and  $\nu$ , or gauge group index  $a$  and  $b$  respectively. Explicitly, they read

$$d_\mu^{ab} = \partial_\mu \delta^{ab} + \Gamma_\mu^{ab} \quad (6.28)$$

$$\mathcal{S}_v^{\mu\nu, ab} = \frac{\kappa^2}{8} \langle [\bar{\Delta}^\mu, \lambda^a][\bar{\Delta}^\nu, \lambda^b] + [\bar{\Delta}^\nu, \lambda^a][\bar{\Delta}^\mu, \lambda^b] \rangle - \frac{\kappa^2}{4} \delta^{\mu\nu} \langle [\bar{\Delta}_\rho, \lambda^a][\bar{\Delta}^\rho, \lambda^a] \rangle \quad (6.29)$$

$$\mathcal{A}_v^{\mu\nu, ab} = \frac{3}{16} i \langle [\lambda^a, \lambda^b](\xi \bar{L}^{\mu\nu} \xi^\dagger + \xi^\dagger \bar{R}^{\mu\nu} \xi) \rangle + \left(\frac{1}{8} + \frac{4\gamma}{3g^2}\right) \langle [\lambda^a, \lambda^b][\bar{\Delta}^\mu, \bar{\Delta}^\nu] \rangle$$

$$\begin{aligned}
& -\frac{\kappa^2}{8} < [\bar{\Delta}^\mu, \lambda^a][\bar{\Delta}^\nu, \lambda^b] - [\bar{\Delta}^\mu, \lambda^a][\bar{\Delta}^\nu, \lambda^b] > . \\
\mathcal{S}_a^{\mu\nu,ab} &= \left(\frac{1}{8\kappa^2} - \frac{4\gamma}{3g_A^2}\right) < [\bar{\Delta}^\mu, \lambda^a][\bar{\Delta}^\nu, \lambda^b] + [\bar{\Delta}^\nu, \lambda^a][\bar{\Delta}^\mu, \lambda^b] > \\
& - \left(\frac{1}{4\kappa^2} - \frac{8\gamma}{3g_A^2}\right) \delta^{\mu\nu} < [\bar{\Delta}_\rho, \lambda^a][\bar{\Delta}^\rho, \lambda^b] > - \frac{8\gamma}{3g_A^2} \delta^{\mu\nu} < \bar{\Delta}_\rho \lambda^a \bar{\Delta}^\rho \lambda^b > \\
& - \frac{4\gamma}{3g_A^2} < \{\lambda^a, \lambda^b\} \{\bar{\Delta}^\mu, \bar{\Delta}^\nu\} > - \frac{2\theta}{g_A^2} \delta^{\mu\nu} < \{\lambda^a, \lambda^b\} (\xi \chi^\dagger \xi + \xi^\dagger \chi \xi^\dagger) >, \tag{6.30}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_a^{\mu\nu,ab} &= \left(\frac{1}{16} + \frac{1}{8\kappa^2} - \frac{2\gamma}{3g_A^2}\right) i < [\lambda^a, \lambda^b] (\xi \bar{L}^{\mu\nu} \xi^\dagger + \xi^\dagger \bar{R}^{\mu\nu} \xi) > + \frac{1}{8} < [\lambda^a, \lambda^b] [\bar{\Delta}^\mu, \bar{\Delta}^\nu] > \\
& - \left(\frac{1}{8\kappa^2} - \frac{4\gamma}{3g_A^2}\right) < [\bar{\Delta}^\mu, \lambda^a][\bar{\Delta}^\nu, \lambda^b] - [\bar{\Delta}^\mu, \lambda^a][\bar{\Delta}^\nu, \lambda^b] > . \tag{6.31}
\end{aligned}$$

and

$$\nabla_{\mu\nu}^{(t)} = -\delta_{\mu\nu}(\partial^2 + m_t^2) \tag{6.32}$$

$$\mathcal{H}_t^{\mu\nu,ab} = \frac{1}{2} \delta^{\mu\nu} (\{\partial^\rho, \Gamma_\rho^{ab}\} + \Gamma_\rho^{ac} \Gamma^{cb}) + \mathcal{S}_t^{\mu\nu,ab} + \mathcal{A}_t^{\mu\nu,ab} \tag{6.33}$$

The modified kinetic terms of vector mesons and axial-vector mesons in Eqs.( 6.27) and ( 6.32) yield their propagators in momentum space as follows

$$\Delta_{\mu\nu}^{(t)}(q) = \frac{1}{q^2 - m_t^2 + i\epsilon} \tag{6.34}$$

This propagator now have a good ultraviolet behavior and chiral symmetry can keep well in follow calculation <sup>8</sup>. The adjoint operators of  $\mathcal{H}_j^{\mu\nu,ab}$  can be obtained from Eq.( 6.32) as follow

$$\mathcal{H}_t^{*\mu\nu,ab} = \mathcal{H}_t^{\nu\mu,ba} \tag{6.35}$$

Up to  $O(p^4)$  there are four kinds of potential one-loop graphs(fig. 8) which involve vector meson or axial-vector meson internal lines only. From propagator of spin-1 mesons in Eq.( 6.34), it can be easily found that quadratic divergence only come from “tadpole” graph(fig. 8-a)). In fact, the effective lagrangian generated by “tadpole” graph is  $O(p^2)$ , Therefore, it is nothing but to modify the free parameters “ $f_0$ ”, “ $B_0$ ” and axial-vector meson mass-dependent parameter “ $\bar{m}_2$ ”. In addition, it is determined by Eq.( 6.34) that much higher order divergences do not appear here. According to previous discussion, the neglected higher order divergences is smaller than logarithmic one due to small square of momentum transfer. Sequentially, we only need to focus our attention on logarithmic divergence and finite terms in our calculation.

Now substituting Eq.( 6.32)-( 6.35) into general formula( 8.2) and performing loop-momentum integral explicitly, we obtain

$$\mathcal{L}_{one-loop}^{(tt)} = \frac{1}{(4\pi)^{D/2}} \left(\frac{\mu^2}{m_j^2}\right)^2 \Gamma(2 - \frac{D}{2}) \left\{ \frac{1}{6} \Gamma_{\mu\nu}^{ab} \Gamma^{\mu\nu,ba} - \mathcal{A}_{t\mu\nu}^{ab} \mathcal{A}_t^{\mu\nu,ba} + \mathcal{S}_{t\mu\nu}^{ab} \mathcal{S}_t^{\mu\nu,ba} \right\} \tag{6.36}$$

---

<sup>8</sup>It is obvious that Eq.( 6.34) is just leading order contribution of Eq.( 6.19) because of small momentum transfer. However, we can not obtain Eq.( 6.34) simply from Eq.( 6.19), since it will break chiral symmetry of effective lagrangian generated by meson loops.



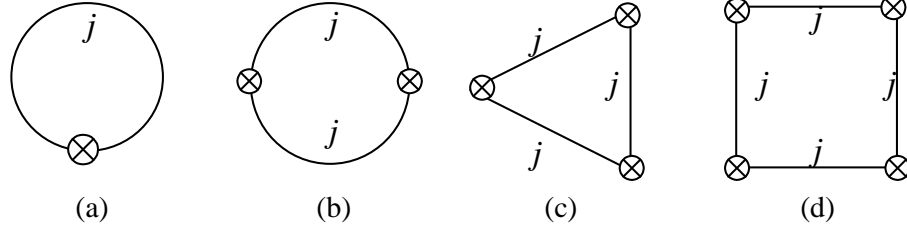


Figure 6.2: Pure one-loop graphs of  $1^-$  or  $1^+$  mesons. Here “ $\otimes$ ” denotes classic external fields and  $j$  denotes internal lines of  $1^-$  or  $1^+$  mesons.

Inserting Eq.( 6.10) and ( 6.28) into ( 6.36) and choosing  $N_f = 3$  we can obtain the effective lagrangian generated by vector meson one-loop as follows

$$\begin{aligned}
\mathcal{L}_{one-loop}^{(vv)} = & \frac{1}{(4\pi)^{D/2}} \left( \frac{\mu^2}{m_v^2} \right)^2 \Gamma(2 - \frac{D}{2}) \{ \frac{19}{32} < \bar{L}_{\mu\nu} \bar{L}^{\mu\nu} + \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} + 2\bar{L}_{\mu\nu} \bar{U}^\dagger \bar{R}^{\mu\nu} \bar{U} > \\
& - \frac{19}{24} < \bar{V}_{\mu\nu} > < \bar{V}^{\mu\nu} > + (\frac{15\gamma}{2g^2} - \frac{1}{2}) i < \bar{L}^{\mu\nu} D_\mu \bar{U}^\dagger D_\nu \bar{U} + \bar{R}^{\mu\nu} D_\mu \bar{U} D_\nu \bar{U}^\dagger > \\
& + (\frac{1}{24} + \frac{\kappa^4}{24} - \frac{25\gamma^2}{9g^4}) (< D_\mu \bar{U}^\dagger D^\mu \bar{U} >^2 + 2 < D_\mu \bar{U}^\dagger D_\nu \bar{U} > < D^\mu \bar{U}^\dagger D^\nu \bar{U} >) \\
& + (\frac{9}{64} \kappa^4 - \frac{3}{8} + \frac{25\gamma^2}{g^4}) < D_\mu \bar{U}^\dagger D^\mu \bar{U} D_\nu \bar{U}^\dagger D^\nu \bar{U} > \} \quad (6.37)
\end{aligned}$$

Similarly, the effective lagrangian generated by axial-vector meson one-loop reads

$$\begin{aligned}
\mathcal{L}_{one-loop}^{(aa)} = & \frac{1}{(4\pi)^{D/2}} \left( \frac{\mu^2}{m_A^2} \right)^2 \Gamma(2 - \frac{D}{2}) \{ A_1 < \bar{L}_{\mu\nu} \bar{L}^{\mu\nu} + \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} + 2\bar{L}_{\mu\nu} \bar{U}^\dagger \bar{R}^{\mu\nu} \bar{U} > \\
& - \frac{4}{3} A_1 < \bar{V}_{\mu\nu} > < \bar{V}^{\mu\nu} > + A_2 i < \bar{L}^{\mu\nu} D_\mu \bar{U}^\dagger D_\nu \bar{U} + \bar{R}^{\mu\nu} D_\mu \bar{U} D_\nu \bar{U}^\dagger > \\
& + A_3 < D_\mu \bar{U}^\dagger D^\mu \bar{U} >^2 + A_4 < D_\mu \bar{U}^\dagger D_\nu \bar{U} > < D^\mu \bar{U}^\dagger D^\nu \bar{U} > \\
& + A_5 < D_\mu \bar{U}^\dagger D^\mu \bar{U} D_\nu \bar{U}^\dagger D^\nu \bar{U} > + A_6 < D_\mu \bar{U}^\dagger D^\mu \bar{U} > < \chi \bar{U}^\dagger + \chi^\dagger \bar{U} > \\
& + A_7 < D_\mu \bar{U}^\dagger D^\mu \bar{U} (\bar{U}^\dagger \chi + \chi^\dagger \bar{U}) > + \frac{1408\theta^2}{9g_A^4} < \chi \bar{U}^\dagger + \chi^\dagger \bar{U} >^2 \\
& + \frac{640\theta^2}{3g_A^4} < \chi \bar{U}^\dagger \chi \bar{U}^\dagger + \chi^\dagger \bar{U} \chi^\dagger \bar{U} > \} \quad (6.38)
\end{aligned}$$

where

$$\begin{aligned}
A_1 &= \frac{27}{32\kappa^4} (1 - \frac{40\gamma}{9g^2})^2 - \frac{1}{4} \\
A_2 &= \frac{9\gamma}{2\kappa^4 g^2} (1 - \frac{40\gamma}{9g^2}) - \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
A_3 &= \frac{1}{16} + \frac{1}{\kappa^4} \left( \frac{15}{128} - \frac{11\gamma}{6g^2} + \frac{1261\gamma^2}{162g^4} \right) \\
A_4 &= \frac{1}{8} + \frac{1}{\kappa^4} \left( \frac{15}{64} - \frac{4\gamma}{3g^2} + \frac{173\gamma^2}{81g^4} \right) \\
A_5 &= -\frac{3}{8} + \frac{1}{\kappa^4} \left( \frac{3}{64} - \frac{2\gamma}{g^2} + \frac{563\gamma^2}{27g^4} \right) \\
A_6 &= \frac{2\theta}{\kappa^4 g^2} \left( \frac{448\gamma}{9g^2} - 3 \right) \\
A_7 &= \frac{6\theta}{\kappa^4 g^2} \left( \frac{256\gamma}{9g^2} - 3 \right)
\end{aligned} \tag{6.39}$$

### 6.3 Mixing loops of vector and axial-vector mesons

Due to discussion in above subsection, the vertices include a quantum field of vector mesons and a quantum field of axial-vector mesons read from lagrangian( 3.20) as follow

$$\begin{aligned}
\mathcal{L}_{(va)} &= v_\mu^a \Omega^{\mu\nu,ab} a_\nu^a \\
\Omega^{\mu\nu,ab} &= B_1 \delta^{\mu\nu} \bar{\Delta}_\rho^{ac} d^{\rho,cb} + \Sigma^{\mu\nu,ab} \\
\Sigma^{\mu\nu,ab} &= -\frac{4\gamma}{3\kappa g^2} (d^\nu \bar{\Delta}^\mu)^{ab} - \frac{\kappa}{2} i < [\lambda^a, \lambda^b] (\xi \bar{L}^{\mu\nu} \xi^\dagger - \xi^\dagger \bar{R}^{\mu\nu} \xi) > \\
&\quad + B_2 < [\lambda^a, \lambda^b] (\xi \chi^\dagger \xi - \xi^\dagger \chi \xi^\dagger) > .
\end{aligned} \tag{6.40}$$

where

$$\begin{aligned}
B_1 &= \frac{1}{\kappa} \left( 1 - \frac{4\gamma}{g^2} \right) & B_2 &= \frac{1-c}{8\kappa} \left( 1 - \frac{16\gamma}{3g^2} \right) + \frac{2\theta_3}{\kappa g^2} \\
\bar{\Delta}_\mu^{ab} &= < \bar{\Delta}_\mu [\lambda^a, \lambda^b] >, & (d_\mu \bar{\Delta}_\nu)^{ab} &= < d_\mu \bar{\Delta}_\nu [\lambda^a, \lambda^b] > .
\end{aligned} \tag{6.41}$$

The adjoint operator of  $\Omega^{\mu\nu,ab}$  is

$$\Omega^{*\mu\nu,ab} = B_1 \delta^{\mu\nu} d^{\rho,bc} \bar{\Delta}_\rho^{ca} + \Sigma^{\mu\nu,ab} \tag{6.42}$$

To  $O(p^4)$ , the vertices ( 6.28)-(6.31) and ( 6.41) generate the following kinds of mixing one-loop graphs(fig. 6.3)

Inserting vertices ( 6.28)-(6.31), ( 6.41) and propagator( 6.34) into general formula ( 8.3), we can perform the integral of loop-momentum explicitly. The effective lagrangian generated by one-loop in fig. 6.3. is

$$\begin{aligned}
\mathcal{L}_{one-loop}^{(va)} &= \frac{1}{(4\pi)^{D/2}} \Gamma(2 - \frac{D}{2}) \int_0^1 dt \left( \frac{\mu^2}{f(t)} \right)^{\epsilon/2} \left\{ \frac{1}{2} \Sigma_{\mu\nu}^{ab} \Sigma^{\mu\nu,ab} + t B_1 \delta^{\mu\nu} \Sigma_{\mu\nu}^{ab} (d_\rho \bar{\Delta}^\rho)^{ba} \right. \\
&\quad - B_1^2 \delta_{\mu\nu} \bar{\Delta}_\rho^{ab} \bar{\Delta}^{\rho,ab} [t \mathcal{S}_V^{\mu\nu,ca} + (1-t) \mathcal{S}_A^{\mu\nu,ca}] + t(1-t) D \frac{4\gamma^2}{9} (d_\mu \bar{\Delta}_\nu)^{ab} (d_\mu \bar{\Delta}_\nu)^{ba} \\
&\quad + \left( \frac{1}{2} - t + t^2 \right) [(d_\mu \bar{\Delta}^\mu)^{ab} (d_\nu \bar{\Delta}^\nu)^{ba} - (d_\mu \bar{\Delta}_\nu)^{ab} (d^\nu \bar{\Delta}^\mu)^{ba}] \\
&\quad \left. + \frac{1}{4} t(1-t) B_1^4 (\delta_{\mu\nu} \delta_{\rho\sigma} + \delta_{\mu\rho} \delta_{\nu\sigma} + \delta_{\mu\sigma} \delta_{\nu\rho}) \bar{\Delta}^{\mu,ab} \bar{\Delta}^{\nu,bc} \bar{\Delta}^{\rho,cd} \bar{\Delta}^{\sigma,da} \right\},
\end{aligned} \tag{6.43}$$

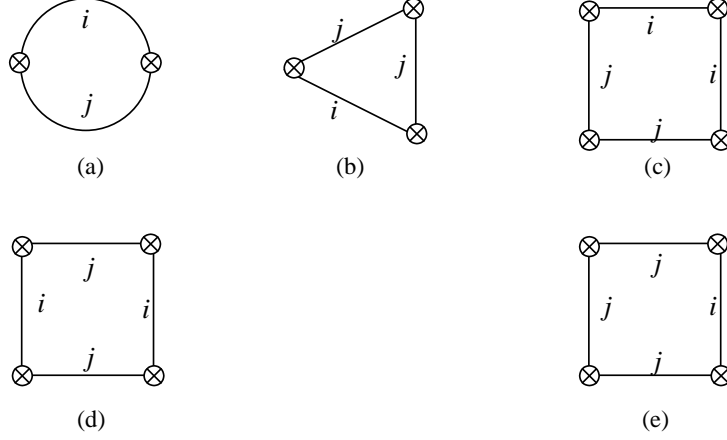


Figure 6.3: Mixing one-loop graphs of  $1^-$  and  $1^+$  mesons. Here “ $\otimes$ ” denotes classic external fields.  $i, j$  denote internal lines of  $1^-$  and  $1^+$  mesons and  $i \neq j$ .

where

$$f(t) = tm_V^2 + (1-t)m_A^2. \quad (6.44)$$

It should be noted that, in Eq.( 6.43), the local  $SU(3)_L \times SU(3)_R$  chiral symmetry is still kept even though we have performed complicated calculation. The explicit form of Eq.( 6.43) with three flavors is

$$\begin{aligned} \mathcal{L}_{one-loop}^{(va)} = & \frac{1}{(4\pi)^{D/2}} \left(\frac{\mu^2}{m^2}\right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \int_0^1 dt \left(\frac{m^2}{f(t)}\right)^{\epsilon/2} \times \\ & \{C_1(t) < \bar{L}_{\mu\nu} \bar{L}^{\mu\nu} + \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} - 2\bar{L}_{\mu\nu} \bar{U}^\dagger \bar{R}^{\mu\nu} \bar{U} > - \frac{4}{3} C_1(t) < \bar{A}_{\mu\nu} > < \bar{A}^{\mu\nu} > \\ & + C_2(t) i < \bar{L}^{\mu\nu} D_\mu \bar{U}^\dagger D_\nu \bar{U} + \bar{R}^{\mu\nu} D_\mu \bar{U} D_\nu \bar{U}^\dagger > + C_3(t) < D_\mu \bar{U}^\dagger D^\mu \bar{U} >^2 \\ & + C_4(t) < D_\mu \bar{U}^\dagger D_\nu \bar{U} > < D^\mu \bar{U}^\dagger D^\nu \bar{U} > + C_5(t) < D_\mu \bar{U}^\dagger D^\mu \bar{U} D_\nu \bar{U}^\dagger D^\nu \bar{U} > \\ & + C_6(t) < D_\mu \bar{U}^\dagger D^\mu \bar{U} > < \chi \bar{U}^\dagger + \chi^\dagger \bar{U} > + 3C_6(t) < D_\mu \bar{U}^\dagger D^\mu \bar{U} (\bar{U}^\dagger \chi + \chi^\dagger \bar{U}) > \\ & + 16B_2^2 < \chi \bar{U}^\dagger - \chi^\dagger \bar{U} >^2 + C_7(t) < \chi \bar{U}^\dagger \chi \bar{U}^\dagger + \chi^\dagger \bar{U} \chi^\dagger \bar{U} > \} \end{aligned} \quad (6.45)$$

where

$$\begin{aligned} C_1(t) &= 3\left(1 - \frac{3\gamma}{g^2} + \frac{8\gamma^2}{9\kappa^2 g^4}\right) - 3B_1^2(t - t^2) \\ C_2(t) &= \frac{8\gamma^2}{3\kappa^2 g^4} + \frac{3}{2} B_1^2(1 - 2t)^2 \\ C_3(t) &= -\frac{2\gamma^2}{3\kappa^2 g^4} - B_1^2\left(\frac{3}{8} + \frac{3}{4\kappa^2} - \frac{32\gamma}{3g_A^2}\right) + tB_1^2\left(\frac{3}{4\kappa^2} + \frac{4\gamma}{g^2} - \frac{32\gamma}{3g_A^2} + \frac{9}{4} B_1^2\right) \end{aligned}$$

$$\begin{aligned}
& -\frac{3}{4}t^2 B_1^2(2-3B_1^2) \\
C_4(t) &= -\frac{4\gamma^2}{3\kappa^2 g^4} - B_1^2\left(\frac{3}{4} + \frac{3}{2\kappa^2} - \frac{16\gamma}{3g_A^2}\right) + tB_1^2\left(\frac{3}{2\kappa^2} + \frac{8\gamma}{g^2} - \frac{64\gamma}{3g_A^2} + \frac{9}{2}B_1^2\right) \\
& -\frac{3}{2}t^2 B_1^2(2-3B_1^2) \\
C_5(t) &= \frac{4\gamma^2}{\kappa^2 g^4} - 3B_1^2\left(\frac{3}{4} + \frac{3}{4\kappa^2} - \frac{32\gamma}{3g_A^2}\right) - 3tB_1^2\left(9 + \frac{9}{4}\kappa^2 - \frac{3}{4\kappa^2} + \frac{32\gamma}{3g_A^2}\right) + 9t^2 B_1^2 \\
C_6(t) &= (1-t)\frac{32\theta}{g_A^2} \\
C_7(t) &= -12[2B_2 + \frac{\gamma}{3\kappa g^2}(1-c)]^2 + 6tB_1(4B_2 - \frac{4\gamma}{3\kappa g^2} + \frac{1-c}{4}) - \frac{3}{2}t^2 B_1^2(1-c) \quad (6.46)
\end{aligned}$$

#### 6.4 Mixing loops of pseudoscalar and spin-1 mesons

The definition of spin-1 meson field ( 2.10) and the field shift ( 3.13) lead to there are no coupling between spin-1 mesons and pseudoscalar fields in  $\mathcal{L}_2^{(0)}$ . It make calculation on meson one-loop be very easily, especially, for calculation on mixing loops of  $0^-$  and  $1^\pm$  mesons.

Since all vertices combining with one spin-1 meson internal line and one pseudoscalar meson internal line come from  $\mathcal{L}_4^{(0)}$ , every vertices is composed of momentum factor  $4 - n_I^{V,A}$ , where  $n_I^{V,A}$  is number of spin-1 meson internal lines attaching to this vertex. Our goal in this paper is to calculate effective lagrangian generated by meson one-loop graphs and expand it up to four powers of external derivatives. Then in a Feynman diagram, momentum powers carried by all internal lines are  $4N_V - 2N_I^{V,A} - 4$ , so that integral of loop-momentum must be proportional to

$$\int \frac{d^4 p}{(2\pi)^4} \frac{q^{4N_V - 2N_I^{V,A} - 4}}{(q^2 + i\epsilon)^{N_I^\phi} (q^2 - m_T^2 + i\epsilon)^{N_I^{V,A}}} \quad (6.47)$$

where  $N_V$  is number of vertices,  $N_I^{V,A}$  is number of all spin-1 meson internal lines and  $N_I^\phi$  is number of all  $0^-$  meson internal lines in the Feynman diagram. Due to relationship for case of one-loop

$$N_I^{V,A} = N_L + N_V - N_I^\phi - 1 = N_V - N_I^\phi,$$

loop-momentum integral( 6.47) is equal to

$$\int \frac{d^4 p}{(2\pi)^4} \frac{q^{2(N_V - 2)}}{(q^2 - m_T^2 + i\epsilon)^{N_V - N_I^\phi}}. \quad (6.48)$$

From the following two aspects we will illustrate the contribution proportioning loop-momentum integral can be omitted: (a) Because  $N_I^\phi \geq 1$  in this kind of mixing loops. the divergences in Eq.( 6.47) is quadratic or higher order. According to discussion in section 6.2, it is smaller than logarithmic divergence due to small square of momentum transfer. (b) Since external momentum is  $O(p^4)$ , this part contribution is equivalent to "tadpole" contribution of spin-1 mesons which is generated by  $\mathcal{L}_6$ . We have point out that in this formalism these interaction with higher order derivatives is not defined. Therefore we have to ignore it here. Sequentially, all contributions from mixing loops of pseudoscalar and spin-1 mesons are omitted in this paper.

## 6.5 Factorization of Divergences

ChQM is a non-renormalizable truncated field theory, since there are not enough counterterms to cancel divergences from meson loops. So that the divergences from meson loops have to be parameterized (or introduce ultra-violet cut-off).

Besides ultra-violet divergences, there are infrared divergences in pure pseudoscalar meson loops. Therefore, we have defined an independent parameter in sect. 6.1 to put the ultra-violet and the infrared divergences into a “bag” simultaneously

$$g_1 = \lim_{\mu_p \rightarrow 0} \frac{1}{(4\pi)^{D/2}} \left( \frac{\mu^2}{\mu_p^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \quad (6.49)$$

In the case of presence of spin-1 mesons, since momentum transfer is smaller than spin-1 meson masses, we like to introduce an ultra-violet cut-off  $\Lambda_M$  to “renormalization” all divergences from spin-1 meson loops approximately.<sup>9</sup>

Explicitly, we define

$$g_2 = \frac{1}{(4\pi)^{D/2}} \left( \frac{\mu^2}{m_V^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \simeq \frac{1}{16\pi^2} \left[ \ln \left( 1 + \frac{\Lambda_M^2}{m_V^2} \right) - \frac{\Lambda_M^2}{\Lambda_M^2 + m_V^2} \right] \quad (6.50)$$

$$g_3 = \frac{1}{(4\pi)^{D/2}} \left( \frac{\mu^2}{m_A^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}) \simeq \frac{1}{16\pi^2} \left[ \ln \left( 1 + \frac{\Lambda_M^2}{m_A^2} \right) - \frac{\Lambda_M^2}{\Lambda_M^2 + m_A^2} \right] \quad (6.51)$$

and

$$\begin{aligned} M &= \frac{1}{(4\pi)^{D/2}} \Gamma(2 - \frac{D}{2}) \int_0^1 dt \left( \frac{\mu^2}{f(t)} \right)^{\epsilon/2} \simeq \frac{1}{16\pi^2} \int_0^1 dt \left[ \ln \left( 1 + \frac{\Lambda_M^2}{f(t)} \right) - \frac{\Lambda_M^2}{\Lambda_M^2 + f(t)} \right] \\ &= \frac{1}{16\pi^2} \left\{ \ln \left( 1 + \frac{\Lambda_M^2}{m_V^2} \right) + \frac{m_A^2}{m_A^2 - m_V^2} \left( \ln \frac{m_V^2}{m_A^2} + \ln \left( \frac{\Lambda_M^2 + m_A^2}{\Lambda_M^2 + m_V^2} \right) \right) \right\} \end{aligned} \quad (6.52)$$

$$\begin{aligned} P &= \frac{1}{(4\pi)^{D/2}} \Gamma(2 - \frac{D}{2}) \int_0^1 dt \cdot t \left( \frac{\mu^2}{f(t)} \right)^{\epsilon/2} \simeq \frac{1}{16\pi^2} \int_0^1 dt \cdot t \left[ \ln \left( 1 + \frac{\Lambda_M^2}{f(t)} \right) - \frac{\Lambda_M^2}{\Lambda_M^2 + f(t)} \right] \\ &= \frac{1}{32\pi^2} \left\{ \ln \left( 1 + \frac{\Lambda_M^2}{m_V^2} \right) + \frac{m_A^4}{(m_A^2 - m_V^2)^2} \ln \frac{m_V^2}{m_A^2} + \frac{\Lambda_M^2}{m_A^2 - m_V^2} \right. \\ &\quad \left. + \frac{m_A^4 - \Lambda_M^4}{(m_A^2 - m_V^2)^2} \ln \left( \frac{\Lambda_M^2 + m_A^2}{\Lambda_M^2 + m_V^2} \right) \right\} \end{aligned} \quad (6.53)$$

$$\begin{aligned} Q &= \frac{1}{(4\pi)^{D/2}} \Gamma(2 - \frac{D}{2}) \int_0^1 dt \cdot t^2 \left( \frac{\mu^2}{f(t)} \right)^{\epsilon/2} \simeq \frac{1}{16\pi^2} \int_0^1 dt \cdot t^2 \left[ \ln \left( 1 + \frac{\Lambda_M^2}{f(t)} \right) - \frac{\Lambda_M^2}{\Lambda_M^2 + f(t)} \right] \\ &= \frac{1}{48\pi^2} \left\{ \ln \left( 1 + \frac{\Lambda_M^2}{m_V^2} \right) + \frac{m_A^6}{(m_A^2 - m_V^2)^3} \ln \frac{m_V^2}{m_A^2} + \frac{\Lambda_M^2}{m_A^2 - m_V^2} \right. \\ &\quad \left. + \frac{\Lambda_M^2 (2\Lambda_M^2 + m_A^2)}{(m_A^2 - m_V^2)^2} - \frac{2\Lambda_M^6 + 3\Lambda_M^4 m_A^2 - m_A^6}{(m_A^2 - m_V^2)^3} \ln \left( \frac{\Lambda_M^2 + m_A^2}{\Lambda_M^2 + m_V^2} \right) \right\} \end{aligned} \quad (6.54)$$

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<sup>9</sup>In fact, similar to sect. 3, it should be different that the cut-offs correspond to pseudoscalar interaction in very low energy and to spin-1 mesons coupling to pseudoscalar in energy scale  $\mu \sim m_\rho$ . According to sect. 6.1 and 6.2, we have evaluated that both of cut-offs are smaller than 700MeV. In this energy region, the difference is not large, so that we take a cut-off here.

Substituting Eqs.( 6.49)-( 6.52) into effective lagrangian generated by meson one-loop, all divergences are canceled.

## 7 Physics Beyond Leading Order of $1/N_c$

In this section we like to summarize the results in sect. 3, sect. 4 and sect.6 and discuss some physics up to the next to leading order of  $1/N_c$  expansion.

### 7.1 Effective Lagrangian

It should be noted that the correction of meson one-loop to the term  $\langle \bar{L}_{\mu\nu}\bar{L}^{\mu\nu} + \bar{R}_{\mu\nu}\bar{R}^{\mu\nu} \rangle$  is nothing since it can be absorbed by  $g_\phi$  and  $g$ . Then up to the next to leading order of  $1/N_c$  expansion, the effective lagrangian with four derivatives can be written

$$\begin{aligned} \mathcal{L}_4 = & \mathcal{L}_{one-loop}^{(\varphi\varphi)} - \frac{\lambda_r(\mu)}{16} \langle L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu} \rangle - \frac{f_1}{8} \langle V_{\mu\nu} \rangle \langle V^{\mu\nu} \rangle \\ & - \frac{f_2}{8} \langle A_{\mu\nu} \rangle \langle A^{\mu\nu} \rangle + R_1 \langle L_{\mu\nu}U^\dagger R^{\mu\nu}U \rangle \\ & - iR_2 \langle D_\mu U D_\nu U^\dagger R^{\mu\nu} + D_\mu U^\dagger D_\nu U L^{\mu\nu} \rangle + R_3 \langle D_\mu U D^\mu U^\dagger \rangle^2 \\ & + R_4 \langle D_\mu U D_\nu U^\dagger \rangle \langle D^\mu U D^\nu U^\dagger \rangle + R_5 \langle D_\mu U D^\mu U^\dagger \rangle \langle D_\nu U D^\nu U^\dagger \rangle \\ & + R_6 \langle D_\mu U D^\mu U^\dagger \rangle \langle \chi U^\dagger + U \chi^\dagger \rangle + R_7 \langle D_\mu U D^\mu U^\dagger (\chi U^\dagger + U \chi^\dagger U) \rangle \\ & + R_8 \langle \chi U^\dagger + U \chi^\dagger \rangle^2 + R_9 \langle \chi U^\dagger - U \chi^\dagger \rangle^2 + R_{10} \langle \chi U^\dagger \chi U^\dagger + \chi^\dagger U \chi^\dagger U \rangle, \end{aligned} \quad (7.1)$$

where

$$\begin{aligned} f_1 &= 8\left(\frac{19}{24}g_2 + \frac{4}{3}A_1g_3\right), \\ f_2 &= 32\left(1 - \frac{3\gamma}{g^2} + \frac{8\gamma^2}{9\kappa^2g^4}\right)M - 32B_1^2(P - Q), \end{aligned} \quad (7.2)$$

and  $\lambda_r(\mu)$ ,  $L_{\mu\nu}$ ,  $R_{\mu\nu}$ ,  $D_\mu U$  and  $D_\mu U^\dagger$  are defined in Eqs.( 3.12), ( 3.21) and ( 3.22). According to sect. 3.1, the constants  $R_i$  ( $i = 1, 2, \dots, 11$ ) contain three different classes of contributions, i.e.,

$$R_i = R_i^{(0)} + R_i^{(g)} + R_i^{(l)}.$$

The contribution from quark loops,  $R_i^{(0)}$ , can be obtained from  $\mathcal{L}_4^{(0)}$ ,

$$\begin{aligned} R_1^{(0)} &= -\frac{\gamma}{6}, & R_2^{(0)} &= \frac{\gamma}{3}, \\ R_3^{(0)} &= \frac{1}{2}R_4^{(0)} = -\frac{1}{4}R_5^{(0)} = \frac{\gamma}{24}, \\ R_7^{(0)} &= \theta_1, & R_{10}^{(0)} &= \theta_2, \\ R_6^{(0)} &= R_8(0) = R_9(0) = 0. \end{aligned} \quad (7.3)$$

The contribution from quark-gluon coupling,  $R_i^{(g)}$ , read from  $\mathcal{L}_1^{(g)}$  as follow

$$\begin{aligned}
R_1^{(g)} &= -\frac{k}{40}, & R_5^{(g)} &= -\frac{k}{40}, \\
R_7^{(g)} &= -\frac{km}{8B_0}, & R_{10}^{(g)} &= \frac{km}{8B_0}(1-c-\frac{2m}{B_0})-\frac{k}{160}(1-c)^2 \\
R_2^{(g)} &= R_3^{(g)} = R_4^{(g)} = R_6^{(g)} = R_8(0) = R_9(0) = 0.
\end{aligned} \tag{7.4}$$

$R_i^{(l)}$  generated by meson one-loop effects read explicitly

$$\begin{aligned}
R_1^{(l)} &= \frac{3}{8}(f_1 - f_2), \\
R_2^{(l)} &= -(\frac{15\gamma}{2g^2} - \frac{1}{2})g_2 - A_2g_3 - \frac{8\gamma^2}{3\kappa^2g^4}M - \frac{3}{2}B_1^2(M - 4P + 4Q), \\
R_3^{(l)} &= (\frac{1}{24} + \frac{\kappa^4}{24} - \frac{25\gamma^2}{9g^4})g_2 + A_3g_3 - [\frac{2\gamma^2}{3\kappa^2g^4} + B_1^2(\frac{3}{8} + \frac{3}{4\kappa^2} - \frac{32\gamma}{3g_A^2})]M \\
&\quad + PB_1^2(\frac{3}{4\kappa^2} + \frac{4\gamma}{g^2} - \frac{32\gamma}{3g_A^2} + \frac{9}{4}B_1^2) - \frac{3}{4}QB_1^2(2 - 3B_1^2), \\
R_4^{(l)} &= (\frac{1}{12} + \frac{\kappa^4}{12} - \frac{50\gamma^2}{9g^4})g_2 + A_4g_3 - [\frac{4\gamma^2}{3\kappa^2g^4} + B_1^2(\frac{3}{4} + \frac{3}{2\kappa^2} - \frac{16\gamma}{3g_A^2})]M \\
&\quad + PB_1^2(\frac{3}{2\kappa^2} + \frac{8\gamma}{g^2} - \frac{64\gamma}{3g_A^2} + \frac{9}{2}B_1^2) - \frac{3}{2}QB_1^2(2 - 3B_1^2), \\
R_5^{(l)} &= (\frac{9}{64}\kappa^4 - \frac{3}{8} + \frac{25\gamma^2}{g^4})g_2 + A_5g_3 + [\frac{4\gamma^2}{\kappa^2g^4} - 3B_1^2(\frac{3}{4} + \frac{3}{4\kappa^2} - \frac{32\gamma}{3g_A^2})]M \\
&\quad - 3PB_1^2(9 + \frac{9}{4}\kappa^2 - \frac{3}{4\kappa^2} + \frac{32\gamma}{3g_A^2}) + 9QB_1^2, \\
R_6^{(l)} &= A_6g_3 + (M - P)\frac{32\theta}{g_A^2}, \\
R_7^{(l)} &= A_7g_3 + 3(M - P)\frac{32\theta}{g_A^2}, \\
R_8^{(l)} &= \frac{1408\theta^2}{9g_A^2}g_3, \\
R_9^{(l)} &= 16MB_2^2, \\
R_{10}^{(l)} &= \frac{640\theta^2}{3g_A^2}g_3 - 12M[2B_2 + \frac{\gamma}{3\kappa g^2}(1-c)]^2 + 6PB_1(4B_2 - \frac{4\gamma}{3\kappa g^2} + \frac{1-c}{4}) \\
&\quad - \frac{3}{2}QB_1^2(1-c).
\end{aligned} \tag{7.5}$$

## 7.2 Low energy limit

The low energy limit of ChQM can be obtained via integrating out the degrees of freedom of spin-1 meson resonances. It means that, at very low energy, the dynamics of spin-1 meson resonances are

replaced by pseudoscalar meson. In Ref.[7], the authors have employed this method to a similar model. We have shown in sect. 3.2 that  $\mathcal{L}^\phi$  is just low energy limit of this model. Since there are no spin-1 mesons coupling to pseudoscalar field which make the classical solutions of spin-1 mesons be  $O(p^3)$  at very low energy. It indicates that physical vector and axial-vector meson fields defined by non-linear realization of G and the field shift ( 3.13) do not contribute to low energy coupling constants  $L_i$  by virtual spin-1 field exchange and virtual spin-1 field vertices contributions. It is different from the discussion in Ref.[7] and Ref.[11]. We must point out that these contributions from virtual spin-1 meson resonances are model-dependent, which depend on definition of physical spin-1 meson resonances. However, the physical results should not be changed by the definition. For Example, as Ref.[6] the definitions spin-1 meson fields( $\mathcal{V}_\mu, \mathcal{A}_\mu$ ) are

$$L_\mu = \mathcal{V}_\mu - \mathcal{A}_\mu, \quad R_\mu = \mathcal{V}_\mu + \mathcal{A}_\mu$$

and diagonalization of  $\mathcal{A}_\mu - \partial_\mu \Phi$  mixing is

$$\mathcal{A}_\mu \longrightarrow \mathcal{A}_\mu - c \partial_\mu \Phi.$$

Recalling the definition of spin-1 meson fields in this paper

$$L_\mu = \xi^\dagger (V_\mu - A_\mu) \xi, \quad R_\mu = \xi (V_\mu + A_\mu) \xi^\dagger,$$

the measures of integral for spin-1 meson fields are the same for two different definitions. Therefore, we obtain the same generating functional of Green's function from two different definitions

$$\begin{aligned} e^{iW[v,a,s,p;U]} &= \frac{1}{N} \int \mathcal{D}V_\mu \mathcal{D}A_\mu \exp \{iI_{\text{eff}}[v, a, s, p; V_\mu, A_\mu, U]\} \\ &= \frac{1}{N} \int \mathcal{D}\mathcal{V}_\mu \mathcal{D}\mathcal{A}_\mu \exp \{iI_{\text{eff}}[v, a, s, p; \mathcal{V}_\mu, \mathcal{A}_\mu, U]\}. \end{aligned}$$

Of course, the definition in this paper has attractive symmetrical properties and is very convenient for our calculation.

The explicit expression of the coupling constants  $L_i$ (besides of  $L_7$ ) read

$$\begin{aligned} L_1 &= \frac{g_\phi^2}{32} c^2 (1 - \frac{c}{2})^2 + \frac{R_2}{2} c (1 - \frac{c}{2}) (1 - c)^2 + R_3 (1 - c)^4 + \frac{3}{64} g_1, \\ L_2 &= \frac{g_\phi^2}{16} c^2 (1 - \frac{c}{2})^2 + R_2 c (1 - \frac{c}{2}) (1 - c)^2 + R_4 (1 - c)^4 + \frac{3}{32} g_1, \\ L_3 &= -\frac{3}{16} g_\phi^2 c^2 (1 - \frac{c}{2})^2 - 3R_2 c (1 - \frac{c}{2}) (1 - c)^2 + R_5 (1 - c)^4, \\ L_4 &= R_6 (1 - c)^2 + \frac{g_1}{16}, \\ L_5 &= R_7 (1 - c)^2 + \frac{3}{16} g_1, \\ L_6 &= R_8 + \frac{11}{288} g_1, \\ L_8 &= R_{10} + \frac{5}{96} g_1, \end{aligned} \tag{7.6}$$



$$\begin{aligned}
L_9 &= \frac{g_\phi^2}{8}c(1 - \frac{c}{2}) + R_2(1 - c)^2 + \frac{g_1}{8}, \\
L_{10} &= -\frac{g_\phi^2}{8}c(1 - \frac{c}{2}) + R_1(1 - c)^2 - \frac{g_1}{8}.
\end{aligned} \tag{7.7}$$

The constants  $L_7$  has been known to get dominant contribution from  $\eta_0$ [9] and this contribution is suppressed by  $1/N_c$  too.  $\eta_0$  participate the dynamics in ChQM via  $\Phi(x) \rightarrow \Phi(x) + \Phi_0(x)(\Phi_0(x) = \frac{1}{\sqrt{3}}\eta_0)$  and  $U(1)$  axial anomaly of QCD,  $\mathcal{L}_\chi \rightarrow \mathcal{L}_\chi - \frac{1}{2}\tau < \Phi_0 >^2$ , where

$$\tau \propto < 0 | (\alpha_s/\pi) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} | 0 > \quad (\tilde{G}^{a\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a) \tag{7.8}$$

relate to gluon anomaly and is a free parameter here. The equation of motion of  $\eta_0$  is

$$(\partial^2 + m_{\eta_0}^2(\tau))\eta_0 = \frac{\sqrt{6}}{8}i < \chi^\dagger U - U^\dagger \chi > . \tag{7.9}$$

The lowest order solution of above equation is  $O(p^2)$ . So that  $L_7^g$  can be obtained simply by  $\eta_0$  exchange via integrating over freedom of  $\eta_0$

$$L_7^g = -\frac{f_\pi^2}{128m_{\eta'}^2}, \tag{7.10}$$

where we have ignored the  $\eta - \eta'$  mixing so that we can input  $m_{\eta_0} \simeq m_{\eta'}$ . The total value of  $L_7$  is

$$L_7 = L_7^g + R_9. \tag{7.11}$$

The latter comes from the mixing loops of vector and axial-vector mesons.

Form effective lagrangian( 7.1), we obtain decay width of  $\phi \rightarrow \pi^+\pi^-$  straightforward via a  $\omega$  exchange

$$\Gamma(\phi \rightarrow \pi^+\pi^-) = \frac{2f_1^2 m_\phi^4}{g^4 m_\omega^4} \Gamma(\omega \rightarrow \pi^+\pi^-). \tag{7.12}$$

Input data  $\Gamma(\omega \rightarrow \pi^+\pi^-) = (185 \pm 10)\text{KeV}$ ,  $\Gamma(\phi \rightarrow \pi^+\pi^-) = (0.35 \pm 0.17)\text{KeV}$  and  $g = 0.39$ , we obtain

$$f_1 \simeq 0.0028 \pm 0.0014.$$

Then we can fit  $\Lambda_M \simeq 550\text{MeV}$  and  $g_2 \simeq 0.47 \times 10^{-3}$ ,  $g_3 \simeq 0.12 \times 10^{-3}$ . It is consistent with our previous discussion for  $\Lambda_M < m_\rho$ .

The above results show that, the contributions yielded by spin-1 meson loops is very small and the important contributions come from pseudoscalar one-loop. Therefore, here we can ignore the former and input  $L_1$  and  $L_9$ (the effective gluon coupling do not contribute to  $L_1$  and  $L_9$ ) to fit  $g_1$  and  $g_\phi$ . The results is  $g_1 = 0.003 \gg g_2, g_3$  and  $g_\phi^2 = 0.096$ .

The numerical results on these low energy coupling constants are in table 1, where we take  $f_0 = f_\pi = 185\text{MeV}$ ,  $m_V = m_\rho = 770\text{MeV}$  and  $m_A \simeq 1150\text{MeV}$  obtained in sect. 3. In addition, to leading order of  $m_u$  and  $m_d$ , and taking  $m_u + m_d \simeq 9\text{MeV}$ [37], we have  $B_0 = \frac{m_\pi^2}{m_u + m_d} \simeq 2\text{GeV}$ . We find that all values of  $L_i$  agree with data well.

	ChPT	leading order	one-loop	gluon correction(k=0.038)	Total
$L_1$	$0.7 \pm 0.3$	0.77	0.14	0	0.91
$L_2$	$1.3 \pm 0.7$	1.54	0.28	0	1.82
$L_3$	$-4.4 \pm 2.5$	-4.48	$0.02^a)$	-0.09	-4.55
$L_4$	$-0.3 \pm 0.5$	0	0.13	0	0.13
$L_5$	$1.4 \pm 0.5$	0.43	0.59	-0.21	0.81
$L_6$	$-0.2 \pm 0.3$	0	0.07	0	0.07
$L_7$	$-0.4 \pm 0.15$	0	$5 \times 10^{-7}$	$-0.4 \pm 0.1^b)$	$-0.4 \pm 0.1$
$L_8$	$0.9 \pm 0.3$	0.38	0.16	0.16	0.70
$L_9$	$6.9 \pm 0.7$	6.1	0.38	0	6.48
$L_{10}$	$-5.2 \pm 0.3$	-5.1	-0.38	-0.30	-5.78

Table 1:  $L_i$  in units of  $10^{-3}$ ,  $\mu = m_\rho$ . a)contribution from spin-1 meson loops.  
b)contribution from gluon anomaly.

In the following we like to fit the parameter  $k$  phenomenologically and check whether it coincides with the result from QCD sum rules. Since quadratic gluon condensate is weakly dependent on scale, we choose a scale-independent quantity,  $L_9 + L_{10}$ , to fit it here. The experimental data require  $L_9 + L_{10} \geq 0.7 \times 10^{-3}$ , if we ignore contribution from spin-1 meson loops, from Eq.( 7.5) we obtain

$$\frac{\gamma}{6}(1-c)^2 - \frac{k}{40}(1-c)^2 \geq 0.7 \times 10^{-3}.$$

So that we have  $k \leq 0.038$ . This value do not conflict with the result from QCD sum rules.

It has been pointed out in sect. 3 that, there are six free parameters to parameterize effective lagrangian generated from quark loops. Up to the next leading order of  $1/N_c$ , four extra free parameters are needed. They are  $k$ ,  $g_1$ ,  $\Lambda_M$  and  $\tau$ . Then up to powers four of derivatives and the next leading order of  $1/N_c$ , the total ten real free parameters determine dynamics with energy scale below axial-vector meson masses completely.

### 7.3 Reexamining chiral sum rules

#### 1) $\rho \rightarrow \pi\pi$ decay and KSRF sum rules

Up to the next leading order of  $1/N_c$ ,  $f_{\rho\pi\pi}$  reads

$$f_{\rho\pi\pi} = \frac{m_\rho^2}{gf_\pi^2} [16R_2(1-c)^2 + 2g^2c(1 - \frac{c}{2})] = 5.92. \quad (7.13)$$

Then width of  $\rho \rightarrow \pi\pi$  decay is yielded as follows

$$\Gamma(\rho \rightarrow \pi\pi) = \frac{f_{\rho\pi\pi}^2}{48\pi} m_\rho (1 - \frac{4m_\pi^2}{m_\rho^2})^{\frac{3}{2}} = 145 MeV. \quad (7.14)$$

It agree with experimental data 150MeV well. The KSRF (I) sum rules[39]

$$g_{\rho\gamma}(m_\rho^2) = \frac{1}{2} f_{\rho\pi\pi} f_\pi^2$$

is the result of current algebra and PCAC.  $g_{\rho\gamma} = \frac{1}{2}gm_\rho^2$  has been obtained in Eq.( 3.29). The KSRF (I) sum rule yield

$$g = f_{\rho\pi\pi} \frac{f_\pi^2}{m_\rho^2} = 0.343.$$

The error bar is about 10%. Considering that error bar between KSRF (I) sum rule and experiment is about 10% too, our result agree with KSRF (I) sum rule well. Furthermore,  $\frac{1}{2}f_\pi^2 f_{\rho\pi\pi}^2 = (772\text{MeV})^2$  yields the KSRF (II) sum rule[39]  $m_\rho^2 = \frac{1}{2}f_\pi^2 f_{\rho\pi\pi}^2$ .

## 2) Axial vector meson mass and Weinberg sum rule

The axial-vector meson mass relation has been given in Eqs.( 3.16) and ( 3.34). In leading order of  $1/N_c$ ,  $m_A = 1150\text{MeV}$  close to prediction by the second Weinberg sum rule  $m_A = \sqrt{2}m_V$ . To the next leading to order of  $1/N_c$ ,  $R_1 = -0.036$  and  $g_A^2 = g^2 + 8R_1$  yield  $m_A = 1230\text{MeV}$  which very close to experimental data  $m_A = 1230 \pm 40\text{MeV}$ .

The first Weinberg sum rule is[28]

$$\frac{f_\rho^2}{m_\rho^2} - \frac{f_a^2}{m_A^2} = \frac{1}{4}f_\pi^2 \quad (7.15)$$

where  $f_\rho$  and  $f_a$  are defined by matrix element of vector and axial-vector currents

$$\begin{aligned} \langle 0 | \bar{\psi} \frac{\tau^i}{2} \gamma_\mu \psi | \rho^{j\lambda} \rangle &= f_\rho \epsilon_\mu^\lambda \delta^{ij}, \\ \langle 0 | \bar{\psi} \frac{\tau^i}{2} \gamma_\mu \gamma_5 \psi | a_1^{j\lambda} \rangle &= f_a \epsilon_\mu^\lambda \delta^{ij}. \end{aligned} \quad (7.16)$$

In this model, these currents are obtained easily through functional differential for corresponding external fields,

$$\begin{aligned} \bar{\psi} \frac{\tau^i}{2} \gamma_\mu \psi &= \frac{\delta \mathcal{L}_\chi}{\delta v_i^\mu}, \\ \bar{\psi} \frac{\tau^i}{2} \gamma_\mu \gamma_5 \psi &= \frac{\delta \mathcal{L}_\chi}{\delta a_i^\mu}, \end{aligned} \quad (7.17)$$

where  $v$  and  $a$  are vector and axial-vector external fields respectively. To replace  $\mathcal{L}_\chi$  in Eq.( 7.17) by effective lagrangian ( 3.15) and normalize vector and axial-vector mesonic fields we obtain

$$\begin{aligned} \bar{\psi} \frac{\tau^i}{2} \gamma_\mu \psi &= -\frac{1}{2}gm_\rho^2 \rho_\mu^i + \dots, \\ \bar{\psi} \frac{\tau^i}{2} \gamma_\mu \gamma_5 \psi &= \left( \frac{8\lambda(\mu)}{g_A} - \frac{1}{2}g_A m_A^2 \right) a_\mu^i + \dots = -\frac{m_2^2}{2g_A} a_\mu^i + \dots, \end{aligned} \quad (7.18)$$

where the mass relation( 3.16) has been used. Comparing Eq.( 7.16) and Eq.( 7.18) we have

$$f_\rho = -\frac{1}{2}gm_\rho^2, \quad f_a = -\frac{m_2^2}{2g_A}. \quad (7.19)$$

Moreover, due to Eqs.( 3.16) and ( 3.34) we have

$$m_2^2 = m_A^2 g_A^2 - 16\lambda(\mu)m^2 = \frac{f_\pi^2}{c} \simeq m_\rho^2 g^2 \quad (7.20)$$

Substituting Eq.( 7.19) into Eq.( 7.15) and using the mass relation ( 7.20) and Eq.( 3.27) we can prove the first Weinberg sum rule is satisfied in ChQM. It is not surprising since the first Weinberg sum rule is derived from chiral symmetry, PCAC and VMD, and the present theory is just a realization of chiral symmetry, PCAC and VMD.

## 8 Summary

It was well known that, in very low energy, ChPT is a rigorous consequence of the symmetry pattern in QCD and its spontaneous breaking. Effective lagrangian of ChPT depends on a number of low-energy coupling constants which can not be determined from the symmetries of the fundamental theory only. As soon as we start going beyond the lowest order, the number of free parameters increases very rapidly, it makes calculation beyond the lowest few order rather impractical. In addition, at energy lying scale between perturbative QCD and ChPT, there is no well-defined method to yield a explicitly convergence expansion. Therefore, instead of ChPT, some phenomenological models are needed to capture the physics between perturbative QCD and ChPT. ChQM is just the one of such models with fewer free parameters. Since the low energy coupling constants,  $L_i(i = 1, 2, \dots, 10)$ , yielded by fewer parameters in ChQM agree with ChPT, it reflects dynamics constraint of these low energy constants.

In ChQM, the dynamics of composite meson fields are generated by the quark loops. Of course, it is better if we can start with a lagrangian which is purely fermionic and hadronic fields are generated by the model itself. However, it is rather impractical since we know nothing about how quarks and anti-quarks are bounded into hadrons. Furthermore, the most ambitious attempts are to find a chirally symmetric solution to the Schwinger-Dyson equations[40]. These methods are typically plagued by instabilities in the solution of the equation. So that the composite meson fields still have to be added explicitly by hand in this paper.

We have presented in this article a systematic treatment of all spin-1 meson resonances  $V$  and  $A$  in framework of ChQM. All possible chiral couplings between spin-1 meson resonances and pseudoscalar fields were derived up to lowest order in the chiral expansion and the next to leading order of  $1/N_c$  expansion. Therefore, it become possible to study low energy physics of mesons in a unified model with finite number of free parameters. It should be noted that the  $A_\mu - \partial_\mu \Phi$  mixing play a crucial role in the model. This mixing is the energy transition caused by quark loops instead of input from symmetry. Because the gap between  $m_A^2 g_A^2$  and  $f_\pi^2$  is not very large( $m_A^2 g_A^2 \simeq 4f_\pi^2$ ), the constant  $c \simeq 0.45$  defined in the field shift( 3.13) play a significant role. It means that it can

not be consistent if we introduce only vector meson resonances but without their chiral partners in ChQM.

Through using method of SVZ sum rules in framework of ChQM, it was shown that ChQM is indeed a phenomenologically successful model at low energy. It is rather difficult to analytically determine the relation between ChQM and various vacuum condensates in SVZ sum rules. However, it can be found that the following basic idea of ChQM is surprisingly similar to one of SVZ sum rules. 1) The mesonic dynamics in ChQM is generated by quark loops (and gluon loops when quark-gluon coupling is included). Correspondingly, in SVZ sum rules, the hadronic properties are studied through parameterizing the effects caused by the vacuum fields. Here, the effects of quark loops in ChQM correspond to various fermion condensate of SVZ sum rules, and the effects of gluon loops in ChQM correspond to various gluon condensate of SVZ sum rules. 2) Physically, for avoiding double counting, we require that the gluon fields must be *soft* so that exchange effects of gluons between quark fields do not create bound states. The similar conclusion is obtained in SVZ sum rules clearly [41], that “*only soft (low frequency) modes are to be retained in the (gluon) condensates*” [42]. A result is yielded directly from the above conclusion, that contribution from the triple gluon condensate  $\langle 0 | g_s^3 GGG | 0 \rangle$  is smaller than one from triple gluon terms in quadratic gluon condensate  $\langle 0 | g_s^2 GG | 0 \rangle$ . The reason is that momentum power counting of the former is higher than one of the latter. This result also coincides with phenomenological analysis previously.

We have shown how to systematically calculate meson one-loop graphs in ChQM. As well as our expectation, the contributions from soft gluon coupling and meson one-loop effects are both smaller than one from quark loops. We must point out that, instead of  $1/N_c$  suppression, the contribution from meson loops are suppressed by “very long-distance” property of meson interaction in meson loops (or scale property). Theoretically, the large  $N_c$  expansion is still an attractive argument [14]. However, since in practical we only take  $N_c = 3$  instead of  $N_c \rightarrow \infty$ , the next to leading order contribution is not suppressed intensively by  $1/N_c$  expansion. Especially, many contributions from meson one-loop are proportional to flavor number  $N_f = 3$ , so that in fact they do not receive any suppression from a theoretically well-defined expansion. We have to be aware that it is a difficulty how the large  $N_c$  argument is applied in case of  $N_c = 3$  for the real world.

There are some conclusions for meson one-loop correction. 1) It is an important feature on meson loops that the square of momentum transfer in meson loops is very small. This feature indicates that the contributions from the “low-frequency” of quantum meson fields are dominant. It is a common conclusion of low energy theory. 2) Although the contribution from meson loops is small, many nontrivial results are yielded. For example,  $L_4$ ,  $L_6$  and some processes forbidden by OZI rules do not vanish here (further studies on OZI forbidden processes will be done elsewhere). 3) Due to large mass gap between pseudoscalar and spin-1 mesons, contribution from one-loop graphs of  $0^-$  mesons is much larger than the one from one-loops of  $1^\pm$  mesons. 4) The calculation on meson one-loop is formal in this paper. There are still some problems that need to be further studied, e.g., imaginary part of  $S$  matrix element appears only via direct calculation on relevant Feynman diagrams. It is important to examine unitarity of  $S$  matrix element and will be found in other papers.

## ACKNOWLEDGMENTS

We would like to thank Prof. D.N. Gao and Dr. J.J. Zhu for their helpful discussion. This work

is partially supported by NSF of China through C. N. Yang and the Grant LWTZ-1298 of Chinese Academy of Science.

## Appendix Momentum-integral Formulas for Meson One-loops

It has been pointed out that the operators in Eqs.( 5.11)-( 5.16),  $\mathcal{H}_j^{AB}(x)$ ,  $\Omega_{ij}^{AB}(x)$  and their adjoint operators include one differential operator at the most. Therefore we set

$$\begin{aligned}\mathcal{H}_j^{AB}(x) &= \mathcal{C}_j^{\mu,AB}(x)\partial_\mu + \dots \\ \mathcal{H}_j^{*AB}(x) &= \mathcal{C}_j^{*\mu,AB}(x)\partial_\mu + \dots \\ \Omega_{ij}^{AB}(x) &= \mathcal{P}_{ij}^{\mu,AB}(x)\partial_\mu + \dots \\ \Omega_{ij}^{*AB}(x) &= \mathcal{P}_{ij}^{*\mu,AB}(x)\partial_\mu + \dots\end{aligned}\tag{8.1}$$

where  $j = \phi(x)$ ,  $\varphi(x)$  and  $\Psi(x)$  are fields. Employing Eq.( 5.16), Eqs.( 5.11)-( 5.13) are rewritten

$$\begin{aligned}i\mathcal{L}_{one-loop}^{(jj)}(x) &= \int \frac{d^D q}{(2\pi)^D} \{ \mathcal{H}_j^{AB}(x, q) \Delta_{AB}(q) \\ &+ \frac{1}{2} \mathcal{H}_j^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) [(\mathcal{H}_j^{CD}(x, q) + \mathcal{H}_j^{*DC}(x, q)) \Delta_{DA}^{(j)}(q) \\ &+ \frac{1}{3} \mathcal{H}_j^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) (\mathcal{H}_j^{CD}(x, q) + \mathcal{H}_j^{*DC}(x, q)) \Delta_{DE}^{(j)}(q - i\partial) \\ &\times (\mathcal{H}_j^{EF}(x, q) + \mathcal{H}_j^{*FE}(x, q)) \Delta_{FA}^{(j)}(q) \\ &+ \frac{1}{4} \mathcal{H}_j^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) (\mathcal{H}_j^{CD}(x, q) + \mathcal{H}_j^{*DC}(x, q)) \Delta_{DE}^{(j)}(q - i\partial) \\ &\times (\mathcal{H}_j^{EF}(x, q) + \mathcal{H}_j^{*FE}(x, q)) \Delta_{FG}^{(j)}(q - i\partial) (\mathcal{H}_j^{GH}(x, q) + \mathcal{H}_j^{*HG}(x, q)) \Delta_{HA}^{(j)}(q) + \dots \},\end{aligned}\tag{8.2}$$

$$\begin{aligned}i\mathcal{L}_{one-loop}^{(ij)}(x) &= \int \frac{d^D q}{(2\pi)^D} \{ \frac{1}{2} \Omega_{ij}^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) \Omega_{ji}^{DC}(x, q) \Delta_{DA}^{(i)}(q) \\ &+ \frac{1}{2} \sum_{k=i,j} \Omega_{ij}^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) \Omega_{ji}^{DC}(x, q) \Delta_{DE}^{(i)}(q - i\partial) (\mathcal{H}_k^{EF}(x, q) + \mathcal{H}_k^{*FE}(x, q)) \Delta_{FA}^{(k)}(q) \\ &+ \frac{1}{4} \Omega_{ij}^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) \Omega_{ji}^{DC}(x, q) \Delta_{DE}^{(i)}(q - i\partial) \Omega_{ij}^{EF}(x, q) \Delta_{FG}^{(j)}(q - i\partial) \Omega_{ji}^{HG}(x, q) \Delta_{HA}^{(i)}(q) \\ &+ \frac{1}{2} \Omega_{ij}^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) (\mathcal{H}_j^{CD}(x, q) + \mathcal{H}_j^{*DC}(x, q)) \Delta_{DE}^{(j)}(q - i\partial) \\ &\times \Omega_{ji}^{FE}(x, q) \Delta_{FG}^{(i)}(q - i\partial) (\mathcal{H}_i^{GH}(x, q) + \mathcal{H}_i^{*HG}(x, q)) \Delta_{HA}^{(i)}(q) \\ &+ \frac{1}{2} \sum_{k=i,j} \Omega_{ij}^{AB}(x, q) \Delta_{BC}^{(j)}(q - i\partial) \Omega_{ji}^{DC}(x, q) \Delta_{DE}^{(i)}(q - i\partial) (\mathcal{H}_k^{EF}(x, q) + \mathcal{H}_k^{*FE}(x, q)) \\ &\times \Delta_{FG}^{(k)}(q - i\partial) (\mathcal{H}_k^{GH}(x, q) + \mathcal{H}_k^{*HG}(x, q)) \Delta_{HA}^{(k)}(q) + \dots \}\end{aligned}\tag{8.3}$$

$$\begin{aligned}
i\mathcal{L}_{one-loop}^{(\phi\Psi\varphi)}(x) &= \int \frac{d^D q}{(2\pi)^D} \Omega_{\phi\varphi}^{AB}(x, q) \Delta_{(\varphi)BC}(q - i\partial) \Omega_{\varphi\Psi}^{CD}(x, q) \Delta_{(\Psi)DE}(q - i\partial) \\
&\quad \times \Omega_{\Psi\phi}^{EF}(x, q) \Delta_{(\phi)FA}(q) + \dots
\end{aligned} \tag{8.4}$$

where

$$\begin{aligned}
\mathcal{H}_j^{AB}(x, q) &= iq_\mu \mathcal{C}_j^{\mu, AB}(x) + \mathcal{H}_j^{AB}(x) \\
\mathcal{H}_j^{*AB}(x, q) &= iq_\mu \mathcal{C}_j^{*\mu, AB}(x) + \mathcal{H}_j^{*AB}(x) \\
\Omega_{ij}^{AB}(x, q) &= iq_\mu \mathcal{P}_{ij}^{\mu, AB}(x) + \Omega_{ij}^{AB}(x) \\
\Omega_{ji}^{AB}(x, q) &= \Omega_{ij}^{*AB}(x) = iq_\mu \mathcal{P}_{*ij}^{\mu, AB}(x) \partial_\mu + \Omega_{ji}^{AB}(x)
\end{aligned} \tag{8.5}$$

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